

# Introduction to mantle convection

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(Based on *Mantle Convection for Geologists* by Geoffrey Davies, from Cambridge University Press. Figures from this text unless otherwise noted.)

With that basic introduction to thermal convection, we are ready to consider the mantle and how it convects. The mantle has two modes of convection, the *plate mode* and the *plume mode*. The plate mode is plate tectonics: a cold, stiff thermal boundary layer on the outside of the planet founders and sinks into the mantle at subduction zones, pulling the surface plate along behind it. Upwellings are passive (that is they are driven by plate spreading, not by buoyancy). The plate mode arises from the strong cooling at the surface, and the weak internal heating of the mantle. It is responsible for the majority of the heat transfer out of the Earth.

The plume mode is associated with narrow, thermal upwellings in the mantle that may be rooted at the core-mantle boundary. It arises because the mantle is heated from below by the core. The plume mode accounts for only a small fraction of the heat transfer out of the Earth.

Mantle convection in both the plate and plume modes leads to cooling of the Earth over time. Simple, globally integrated energy balances can allow us to formulate and test hypotheses about the thermal history of the Earth.

## 1 The fluid mantle

We know the mantle is solid: seismic shear waves pass through it efficiently, traversing the interior of the Earth multiple times. The stresses associated with these waves are sustained for seconds, at most. When the mantle is subjected to longer-term stresses, it flows like a fluid: the rate of deformation is proportional to the size of the stress. We know this from glacial isostatic rebound.

**Glacial isostatic rebound and mantle viscosity** The ice sheets that covered Fennoscandia during the last ice age melted at about 11,000 years before present. Since then, relative sea-level in those regions has been falling, indicating a rebound of the land surface. This drop in sea-level is approximately exponential, with a time-constant of 4.6 ka. We can use this information to approximate the viscosity of the mantle. Our analysis will equate the isostatic force causing rebound with the viscous resistance force of mantle flow back into the area.

Isostasy requires that pressure is constant at a depth  $D$  in the mantle. A column of rock that was unaffected by the ice sheet has pressure  $\rho_m g D$  at its base. In the affected area, there is a depression of depth  $d$  left by the ice after it melts. Suppose this depression is filled with water. Then it has pressure  $g(\rho_w d + \rho_m(D - d))$  at its base. The pressure difference between these two columns of rock is thus

$$\Delta P = g\rho_m D - g(\rho_w d + \rho_m(D - d)), \quad (1)$$

$$= g(\rho_m - \rho_w)d. \quad (2)$$

We can approximate the force arising from this pressure difference by multiplying by the area over which it is applied. This area is roughly a circle with radius  $R$  and hence

$$F_d = \pi R^2 g(\rho_m - \rho_w)d. \quad (3)$$

This driving force is resisted by a viscous force in the mantle, as mantle rock flows into the area that is rebounding. We can assume that the effect of the pressure gradient is felt over a distance approximately equal to the radius of the ice sheet. If the flow has a speed that is roughly  $v$  then a representative velocity gradient is  $v/R$ . The viscous resisting stress is then approximated as

$$\tau_r = 2\mu \left( \frac{v}{2R} \right) = \mu v/R. \quad (4)$$

Again we multiply by the area over which the stress is applied to get the total force as

$$F_r = \pi R^2 \mu v/R. \quad (5)$$

Since the mantle has no appreciable momentum, the forces must balance:  $F_d + F_r = 0$ . This gives

$$\pi R^2 g(\rho_m - \rho_w)d - \pi R^2 \mu v/R = 0, \quad (6)$$

where the negative sign comes from the fact that the forces are in opposite directions. We can rearrange to give

$$v = g(\rho_m - \rho_w)Rd/\mu. \quad (7)$$

The rate of flow  $v$  must be about equal to the rate of change of  $d$ , so we can write this as

$$\frac{dd}{dt} = -\frac{gR\Delta\rho}{\mu}d. \quad (8)$$

This differential equation has the solution

$$d(t) = d_0 e^{-t/\tau_0}, \quad (9)$$

where  $\tau_0 = \mu/(gR\Delta\rho)$ . Rearranging gives  $\mu = \tau_0 gR\Delta\rho$ . Using values  $g = 10 \text{ m/sec}^2$ ,  $R = 1000 \text{ km}$ ,  $\Delta\rho = 2300 \text{ kg/m}^3$ , and  $\tau_0$  from [Figure 1](#), this gives  $\mu \approx 10^{21} \text{ Pa sec}$ . This is a very large number, which is why strain-rates in the mantle are very small.

So we have used glacial isostatic rebound to (a) demonstrate that the mantle flows over geological time, and (b) obtain an approximate value for the viscosity of the mantle.

**Temperature dependence of viscosity** Our estimate of  $\mu$  actually pertains to the hot interior of the mantle, rather than the cold lithosphere above. The viscosity of the whole mantle over a range of temperatures can be described by an *Arrhenius law*

$$\mu = \mu_r \exp \left[ \frac{E^*}{R_g} \left( \frac{1}{T} - \frac{1}{T_r} \right) \right], \quad (10)$$

where  $\mu_r$  and  $T_r$  are reference values,  $E^*$  is the activation energy (J/mole), and  $R_g$  is the universal gas constant. When  $T \approx T_r$ , the exponential is approximately one, and the viscosity is approximately equal to the reference viscosity. When  $T$  is much

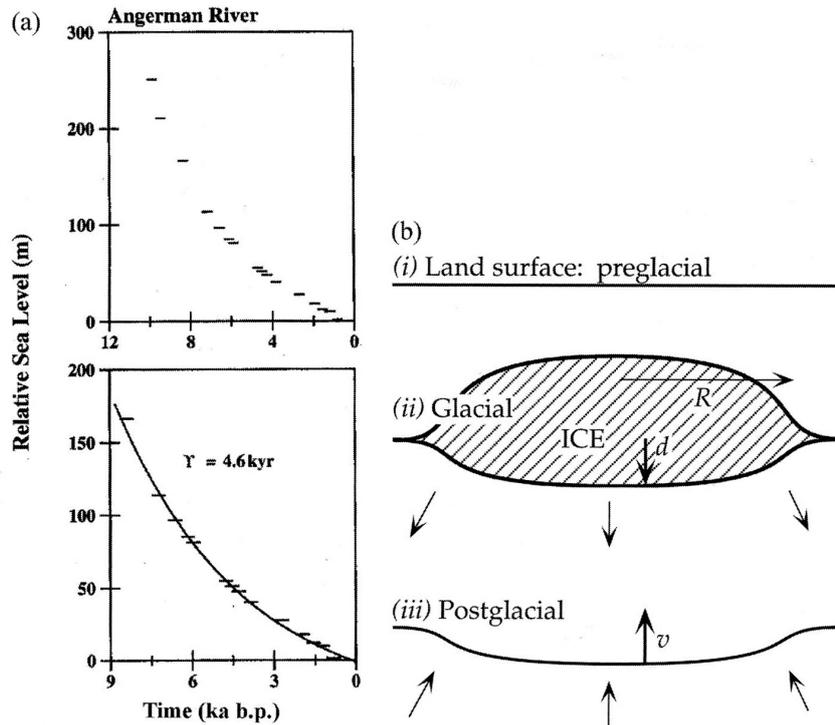


Figure 1: Isostatic rebound of Fennoscandia. (a) shows observational data of relative sea level versus time, with an exponential fit. (b) shows a schematic diagram of the preglacial, synglacial, and postglacial state of the land surface.

smaller than  $T_r$ , the argument to the exponential is large, and hence the viscosity is much larger than  $\mu_r$ .

The temperature dependence of viscosity has an important effect on mantle convection. Remember that  $Ra \propto \mu^{-1}$  and  $Nu \propto Ra^{1/3}$ . If the temperature of the mantle is high, the viscosity is low, and the Rayleigh number is large. This means more vigorous convection, larger Nusselt number, and higher heat flux. The heat flux cools the mantle and causes the viscosity to increase.

## 2 The plate mode

We now know that the mantle is fluid with a large but finite viscosity. We know that plates move and subduct. We know that the mantle convects, and that it is internally heated by radioactive decay. *Is plate tectonics the surface expression of mantle convection?*

**Simple force balance on oceanic plate** To show that plate tectonics is mantle convection, we can demonstrate the plate motions, to leading order, result from the balance of the negative buoyancy of sinking plates and the viscous resistance of mantle flow. In simple mathematical terms,

$$F_B + F_r = 0, \quad (11)$$

where  $F_B$  is the buoyancy force and  $F_r$  is the resisting force.

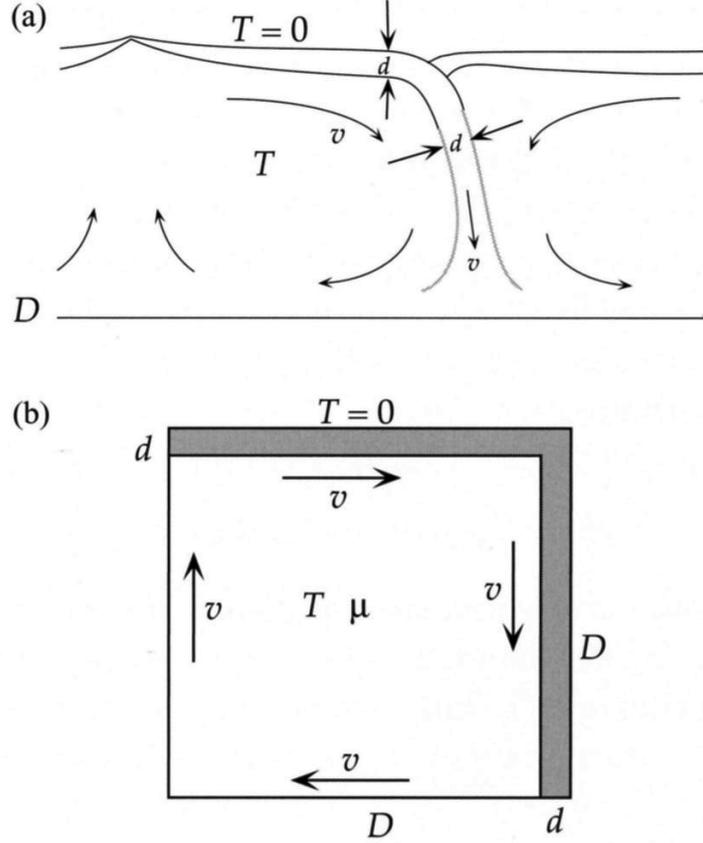


Figure 2: (a) Schematic diagram of the plate mode of convection. (b) Schematic diagram of a simple convection cell driven by sinking of a cold plate into the mantle (from Davies' *Mantle Convection for Geologists*.)

We first turn our attention to  $F_B$ . We'll assume that buoyancy is entirely thermal in origin; that the oceanic lithosphere and the asthenosphere have the same composition. We'll consider a cross-section of a subducting plate, as shown in Figure 2, and all of the equations that we write will be per unit length into the plane of the cross-section. According to Archimedes principle, the buoyancy force (per unit length) is then given by the product of the density difference and the area of the plate in cross-section,

$$F_B = g\rho_0\alpha\Delta T Dd, \quad (12)$$

where  $Dd$  represents the thickness of the plate times its length in the mantle. The temperature difference  $\Delta T$  can be approximated if we assume that the plate has an average temperature that is the mean of the mantle temperature and the surface temperature, so about  $T/2$ . Then  $\Delta T = T - T/2 = T/2$  and we have

$$F_B = g\rho_0\alpha T Dd/2. \quad (13)$$

Because the plate is colder than the mantle, it is negatively buoyant, and hence this force drives it downward.

The resisting force comes from the viscous stress on the subducting plate and the ocean-floor plate. Suppose they are moving with speed  $v$  through a mantle with viscosity  $\mu$ . This creates a shear in the mantle between the plate and the conjugate mantle return flow  $-v$ . These are separated by an approximate distance  $D$  and hence the shear stress is

$$\tau_r = 4\mu v/D, \quad (14)$$

where we get  $2\mu v/D$  for the subducted slab, and the same again for the flat-lying slab. The stress acts on an area  $D$  per unit length of the plate and the resisting force is given by the product of the stress and the area

$$F_r = D4\mu v/D = 4\mu v. \quad (15)$$

Noting that these forces act in opposite directions and plugging them into equation (11) gives

$$-g\rho_0\alpha T D d/2 + 4\mu v = 0, \quad (16)$$

and solving for  $v$  gives

$$v = g\rho_0\alpha T D d/6\mu. \quad (17)$$

To compute the plate thickness  $d$ , we can use the conductive cooling of a plate. If the plate moves a distance  $D$  with a speed  $v$ , the time taken is  $D/v$  and the plate thickness is approximately

$$d = \sqrt{4\kappa D/v}. \quad (18)$$

Substituting (18) into (17) and rearranging gives

$$v = D \left( \frac{g\rho_0\alpha T \sqrt{\kappa}}{3\mu} \right)^{2/3} \quad (19)$$

Taking  $T = 1300^\circ\text{C}$ ,  $D = 3000$  km (the height of the mantle),  $\mu = 10^{22}$  Pa sec (the viscosity of the lower mantle) gives a velocity of about 7 cm/year, which is near the average observed plate motion.

This analysis shows that by thinking of plate tectonics as a convective process driven by the negative buoyancy of subducted plates, we can account for the approximate rate of plate motions at the surface. This does not prove that plates move because of convective forces, nor is this a complete theory of the forces involved in plate tectonics. It turns out that in much more sophisticated models, however, this balance of forces plays a dominant role.

**The yield stress and plate tectonics** If mantle viscosity depended on temperature as equation (10), there would be no plate tectonics because the lithosphere would be too stiff to subduct. The rheology of rock is actually much more complicated, though, and one complication is that it possesses a yield stress. This is a level of stress below which the material is rigid, and above which it flows with little resistance. A yield stress allows plates to bend and break, and hence allows for initiation of subduction or rifting.

**Heat flow due to mantle convection** We have seen that plates, and especially oceanic plates, are part of mantle convection, not just a passive rider on top of it. They are the cold surface boundary layer through which the Earth loses heat by conduction. How much heat does the plate mode remove from the Earth, and what fraction of the total is due to the plate mode of convection?

Let's assume that just before subduction, the lithosphere is 100 Ma old and has a thickness of  $d = 100$  km. It has an average temperature that is about half the difference between the mantle temperature ( $\sim 1300^\circ\text{C}$ ) and the surface temperature, so about  $650^\circ\text{C}$ . The amount of heat lost from the lithosphere between when it left a mid-ocean ridge axis and when it arrives at the subduction zone is

$$\Delta H = \rho d c_P \Delta T, \quad (20)$$

where  $c_P$  is the specific heat capacity.

Sea-floor spreading creates about  $3 \text{ km}^2$  of new ocean floor each year; subduction removes a similar amount. Hence the total rate of heat loss by cooling the oceanic lithosphere is approximately

$$Q = S \Delta H = S \rho d c_P T_m / 2, \quad (21)$$

where  $S$  is the rate of subduction of sea-floor area. Plugging in typical numbers gives about  $21 \times 10^{12}$  Watts, or 21 TW. This estimate is in fact off by about 50%—the actual heat flow out of the ocean basins is about 30 TW.

The total heat flow out of the Earth is 41 TW, so about 75% of this heat flow can be attributed to the plate mode of convection. The oceanic heat flow is about 90% of the heat flow coming out of the mantle.

### 3 The plume mode

For Rayleigh-Benard convection, we saw that there are both cold downwelling currents and hot upwelling currents. The hot upwellings are called plumes. You are already aware that there are plumes in the mantle. This is (probably) because the mantle is not only heated by radioactive decay, but also has a basal heat flux from the hotter core. Secular cooling of the core produces a thermal boundary layer at the base of the mantle (as shown in [Figure 3](#)), which gives rise to thermal plumes (chemical buoyancy may also be important, but we neglect that here). In this section, our goal is to understand the physics of an isolated plume.

Hawaii will be our focus because it is far from a mid-ocean ridge (unlike Iceland) and because it has a relatively large flux and a long history.

**Hotspot swells as a constraint on plume flux** The Hawaiian volcanic chain is associated with a broad swell in the sea floor. The swell is up to 1 km high and about 1000 km wide. There is observational evidence that the crust in this swell is not anomalous; the swell is supported by the buoyant material beneath the lithosphere. Because the Pacific plate is moving while the hot-spot remains stationary, there must be a continuous flux of buoyant rock up the plume conduit to maintain the swell.

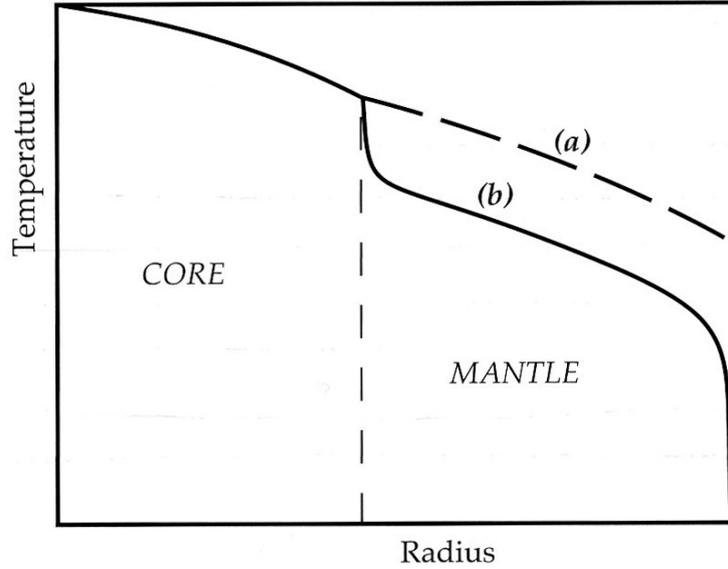


Figure 3: Schematic temperature profiles through the Earth. Profile (a) shows the temperature shortly after accretion and segregation of the core. The mantle is still very hot, and there is no temperature difference between the core and the mantle. Heat would then be lost from the mantle, which would cool as a result. Later the profile would look like (b). Note the thermal boundary layer at the base of the mantle. This could provide a site for formation of hot plumes.

To compute the buoyancy flux of a plume, we consider a simple model of a plume conduit as a vertical cylinder with radius  $r$  and average upwelling speed  $u$ . We consider the short time interval  $\Delta t$  during which the rock in the plume moves, on average, a distance of  $u\Delta t$ . The volume-rate of flow is then

$$\frac{V}{\Delta t} = \pi r^2 u. \quad (22)$$

And we recall that buoyancy is the gravitational force due to a deficit of density,

$$B = g\Delta\rho V. \quad (23)$$

This is the buoyancy of the material that has moved up the plume in a time  $\Delta t$ . Their buoyancy flow rate is the rate at which buoyancy ascends the conduit,

$$b = g\Delta\rho\pi r^2 u. \quad (24)$$

This buoyancy rate can be related to the size of the hotspot swell.

We again consider a balance of forces to obtain an equation. The swell has an excess weight that is due to displacing ocean water with denser rock. Since the plate is moving over the plume, new swell is constantly created as the buoyancy flux elevates the sea floor. If the Pacific plate travels a distance of  $v\delta t = 100$  mm per year and the swell has a height of  $h = 1$  km and width of  $w = 1000$  km, then the downward force of one year's worth of new swell is

$$F_W = g(\rho_m - \rho_w)wvh\delta t, \quad (25)$$

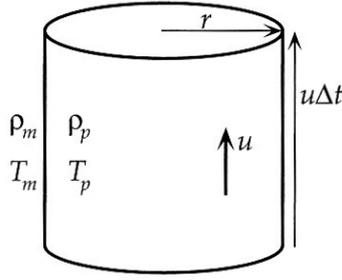


Figure 4: Idealised section of a plume as a vertical cylinder.

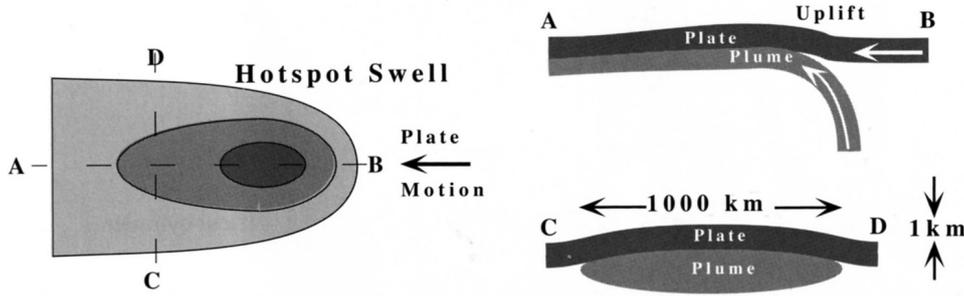


Figure 5: Sketch of a hotspot swell like that of Hawaii in map view (left) and two cross sections.

where the density difference is the mantle density minus the density of water. This must balance the buoyancy force of one year of plume flux,  $F_b = b\delta t$ , which acts in the upward direction. Equating the two gives

$$b = g(\rho_m - \rho_w)wvh. \quad (26)$$

Using values quoted above we obtain  $b = 7 \times 10^4$  N/sec for Hawaii. This is a somewhat abstract quantity, so let's go further. Combine equation (29) with (22) and solve for the volumetric flow rate to obtain

$$\phi = \frac{b}{g\rho_m\alpha\Delta T}. \quad (27)$$

The petrology of Hawaiian lavas indicates a peak temperature of about 300 K above that of normal mantle. Substituting this and other values gives  $\phi = 7.4$  km<sup>3</sup>/yr.

The mean flow rate up the conduit is given as  $u = \phi/\pi r^2$ . Taking a conduit diameter of 100 km for Hawaii (based on the region of active volcanism), we obtain an upwelling speed of nearly 1 m/yr, about 10 times faster than the fastest plate speed!

**Heat transport by plumes** We can relate the buoyancy rate to a heat flow rate by assuming that the buoyancy is entirely thermal in origin. The plume has a temperature  $T_p$ , which is higher than the mantle temperature  $T_m$ . The difference in density is

$$\rho_p - \rho_m = -\rho_m\alpha\Delta T, \quad (28)$$

and so the buoyancy rate can be rewritten as

$$b = \pi r^2 u g \rho_m \alpha \Delta T. \quad (29)$$

Recall that heat content per unit volume is given by  $H = \rho c_P \Delta T$ , where  $c_P$  is the specific heat capacity. The rate at which heat flows up the conduit is then given by

$$Q = \rho_p c_P \Delta T \pi r^2 u, \quad (30)$$

where  $\pi r^2 u$  is the volume flow rate of rock. Making the Boussinesq approximation  $\rho_p \approx \rho_m$ , we can use equation (29) to eliminate  $\rho_p \Delta T \pi r^2 u$  from (30) to obtain

$$Q = \frac{c_P b}{g \alpha}. \quad (31)$$

Hence we have related the heat flow rate up a plume conduit to the buoyancy flow rate with a very simple formula, depending only on three known parameters. Note that we don't need to know  $\Delta T$ ,  $u$ , or  $r$ ! Using standard values we obtain  $Q = 0.2$  TW. This is only about 0.5% of the global heat flow.

The total heat flow of all mantle plumes is only about 2.3 TW, which is about 6% of the global total heat flow. The plate mode of convection accounts for 90% of the heat coming out of the mantle, while the contribution from plumes is much less. This suggests that plumes are a secondary form of convection in the mantle, which is fairly obvious if you look at global topography and heat flow data.

**Plume origins and dynamics** We can think of plumes as originating with the instability of a less dense layer underlying a more dense layer. A small perturbation to the interface can lead to its exponential growth. This perturbation becomes a plume and rises through the dense material above.

The structure of the plume has two parts: the head and the tail. The head is an approximately spherical blob of buoyant fluid. Its size must be large enough for the buoyancy force to overcome viscous resistance. At the same time, it is assimilating surrounding material (and losing heat) by viscous *entrainment*.

The tail of the plume is an approximately cylindrical conduit of buoyant rock that connects the plume to the boundary layer where it originated. Buoyancy drives a flow up the tail. The excess temperature in the tail of the plume reduces the viscosity within it. Lower plume viscosity means that the same flow rate is accommodated by a smaller radius.

**Flow in a plume tail** To model the flow of rock up the tail of a plume, we consider a cylindrical conduit with fixed walls. Within the conduit there is a balance between buoyancy forces (which we assume to be purely thermal) and viscous resistance force.

For a segment of the cylinder with radius  $a$  height  $\delta z$ , the buoyancy force within a radius  $r \leq a$  is

$$F_B = -g \Delta \rho \pi r^2 \delta z. \quad (32)$$

The minus sign cancels the negative in  $\Delta \rho$  to give a positive (upward) buoyancy.

To compute the viscous resistance, in this case, we can use calculus rather than scaling analysis. The viscous resistance is proportional to the gradient in the speed

of the flow,  $dv/dr$ , to give a shear stress that is  $\tau = \mu dv/dr$ . The stress acts over the internal area of the control volume,  $2\pi r\delta z$ . Thus the viscous resistance is

$$F_R = -2\pi r\delta z\mu \frac{dv}{dr}. \quad (33)$$

Equating  $F_B$  with  $F_R$ , integrating once in  $r$  and using the boundary condition  $v = 0$  at  $r = 1$  gives

$$v = -\frac{g\Delta\rho}{4\mu}(r^2 - a^2). \quad (34)$$

We can integrate this equation to obtain the volumetric flow rate over the cross-section of the cylinder. To do so, we must integrate infinitesimal rings of flow, each with an area given by  $dA = 2\pi r dr$ . Hence we have

$$\begin{aligned} \phi &= -\int_0^a 2\pi r \frac{g\Delta\rho}{4\mu}(r^2 - a^2), \\ &= \left(\frac{\pi g\Delta\rho}{8\mu}\right) a^4. \end{aligned} \quad (35)$$

Equation (35) shows that by doubling the conduit radius  $a$ , the volumetric flow rate increases by a factor of  $2^4 = 16$ . This also means that large changes in the flow rate are accommodated by only small changes in the conduit radius. The largest hotspot (Hawaii) has an estimated flow rate of about 10 larger than the smallest; all other parameters being equal, this means the different in conduit radius is less than a factor of two. The seismic detectability of these two plumes is about the same.

We can use our estimated volume flow rate from equation (27) in equation (35) with  $\Delta\rho = \rho\alpha\Delta T$  to solve for the conduit radius  $a$ . Using a plume viscosity of  $10^{18}$  Pa-sec gives a radius  $a \approx 50$  km. Because  $a \propto \mu^{-1/4}$ , a factor-of-ten uncertainty in  $\mu$  translates to a factor-of-two uncertainty in  $a$ . Hence we conclude that plume conduits should be about 50–100 km in radius in the upper mantle.

**Ascent of plume heads** Experiments and numerical simulations tell us that we can approximate the head of a plume as a buoyant sphere. The rate of rise is then given by Stokes law for a spherical particle in a viscous fluid. This involves a balance between buoyancy

$$F_B = 4\pi r^3 g\Delta\rho/3 \quad (36)$$

and Stokes drag

$$F_R = 4\pi r\mu v. \quad (37)$$

Requiring these to balance yields

$$v = g\Delta\rho r^2/3\mu. \quad (38)$$

Using values appropriate to the lower mantle, a plume head with a radius of 500 km would rise at a rate of about 4 cm/yr. In the upper mantle the rise speed will be larger. Figure 6 shows a time-series of results from an axisymmetric model of a plume ascending from a thermal boundary layer. The black dots are passive markers that were initially in the thermal boundary layer.

Note that the rate of flow that we calculated for the conduit ( $\sim 1$  m/yr) is much larger than the rate of rise of the head. We should therefore expect that the head of the plume is growing with time, by this mechanism and by thermal entrainment.

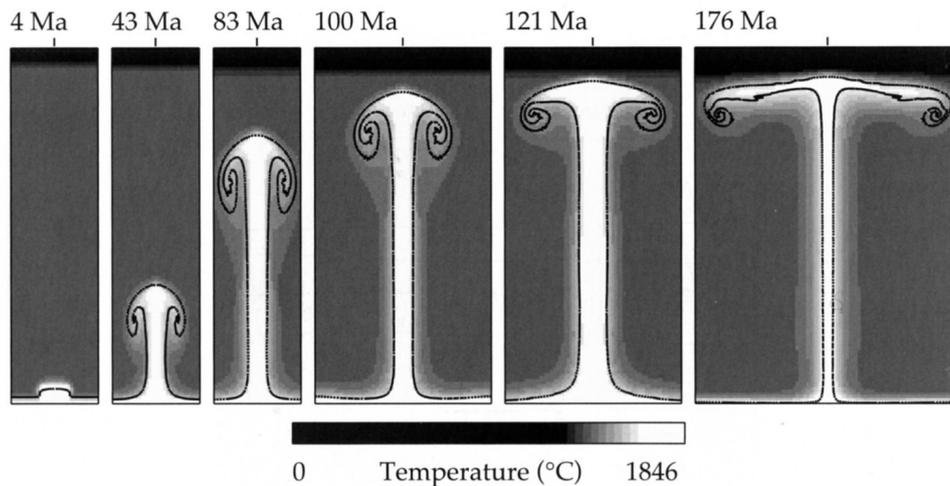


Figure 6: Results from a axisymmetric numerical model in which a plume grows from a thermal boundary layer.

**Thermal entrainment** The complex flow within the head of a plume leads to entrainment of ambient mantle, as shown in Figure 6. Note how the material that was initially in the boundary layer has been stirred into the head of the plume. Thermal diffusion out of the plume head warms the surrounding mantle in another thermal boundary layer. Both entrainment and thermal diffusion cause the plume head to grow with time.

## 4 Overview

**Plumes and plates** Mantle convection is driven, in part, by cooling from above, and in part by heating from below. Why, then, are the downwellings (subducted plates) and the upwellings (plumes) so different in shape and dynamics? What breaks the symmetry between the two? The difference is their viscosity. Plates are cold and behave as rigid units with deformation localised into narrow shear zones. Plumes are hot and flow viscously, with a lower viscosity than their surroundings.

**Layered convection versus whole mantle convection** There is a long-standing debate about whether the mantle convects in two separate layers, or as a single whole. Early arguments stated that the viscosity of the lower mantle (below 660 km) is too large for convection to occur and that geochemical observations suggested the existence of a *primitive reservoir* of primordial material that has been geochemically isolated over the age of the Earth (because it did not participate in convection). Both of these lines of evidence have proven to be incorrect.

The most important evidence against layered convection is images from seismic tomography (Figure 7) that show subducted slabs descending into the lower mantle. Despite their low resolution, these images indicate that slabs penetrate the 660 km boundary. The mass flux of slabs into the lower mantle precludes a persistent chemical isolation of the lower mantle. Consider a simple box model: the lower

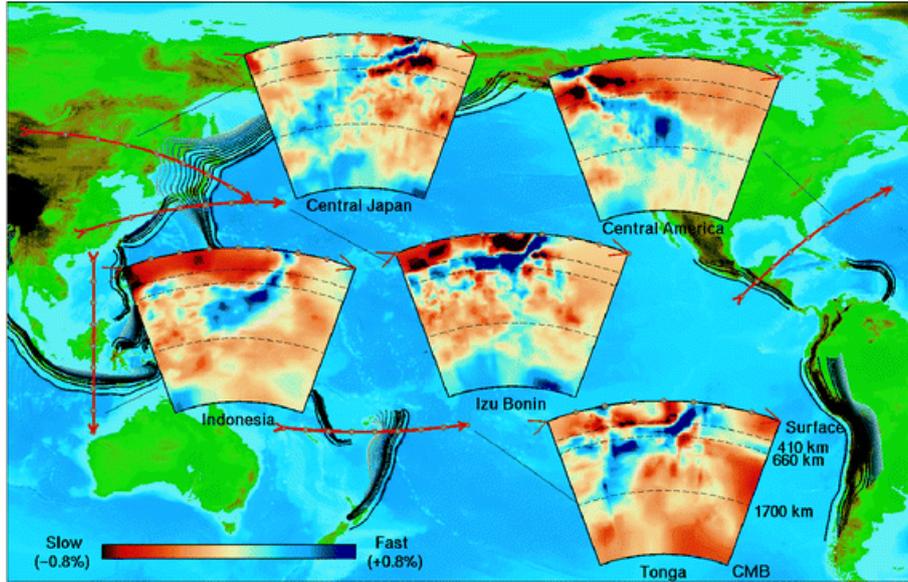


Figure 7: Subduction tomography showing slabs penetrating into the lower mantle. (From Stern, R.J., Subduction Zones, *Rev. Geophys.* 2002)

mantle reservoir contains about  $2.6 \times 10^{24}$  kg of rock. The subducted rate of addition of mass into this reservoir is about  $3 \text{ km}^2/\text{yr}$  subducted  $\times$  100 km plate thickness  $\times$   $3300 \text{ kg/m}^3$  density gives about  $10^{15}$  kg/yr. The residence time of rock in the lower mantle is therefore about 2.6 billion years. Since plate tectonics seems to have operated over much of the history of the Earth, it is very unlikely that there exists a large, pristine reservoir of primitive mantle rock—the lower mantle cannot be chemically isolated from the upper mantle.

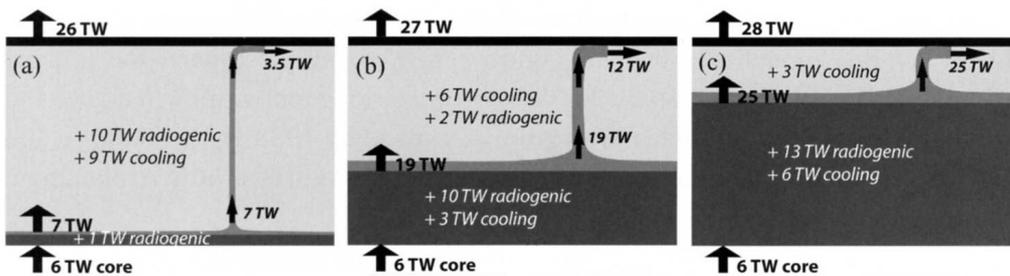


Figure 8: Heat budgets for three models of mantle convection. (a) Full-mantle convection. (b) With a moderately thick lower mantle. (c) With a very thick lower mantle.

There is a second argument against a layered mantle with a lower mantle containing primitive rocks. A deep layer that doesn't participate in convection would have larger internal heating than the upper mantle. This heating, plus the core heat flow would raise its temperature. A thermal boundary layer would develop above it, in the upper mantle, and this would give rise to large plumes, which would have a topographic signature on the sea floor. Such plumes are not observed!

## 5 Energy-balance models

The discussion above has shown that we can consider the mantle to be a single, convecting system that is driven by heating from the core, heating from radioactive decay, and cooling to the atmosphere. We have approximately determined the balance of these rates at the present. What was their balance in the past? Was convection more or less vigorous? We can investigate this with energy-balance models that take current rates and extrapolate backward in time. Fundamentally this means that we apply the first law of thermodynamics to the entire mantle:

$$\text{rate of change of heat content} = \text{rate of heat input} + \text{rate of heat loss.} \quad (39)$$

If the rate of heat loss from the mantle exceeds the rate of heat input, then the mantle is losing energy and cooling down. If the mantle is currently cooling, then we can run the equation backward to determine the heat content (and temperature) of the mantle in the past.

To see this in more detail, we write equation (39) in terms of temperature and rates of heat flow. Recall that changes in internal energy are related to changes in temperature by  $dU = MCdT$ , where  $M$  is the mass of the system and  $C$  is its heat capacity. This can be divided by  $dt$  and rearranged to give

$$\frac{dT}{dt} = \frac{1}{MC} \frac{dU}{dt}, \quad (40)$$

the rate of change of temperature is proportional to the rate of change of internal energy. By the first law, the rate of change of internal energy is related to additions and subtractions of heat, hence we can write

$$\frac{dT}{dt} = \frac{1}{MC} (Q_R + Q_C - Q_S). \quad (41)$$

$Q_R$  (radioactive heating),  $Q_C$  (heat from the core), and  $Q_S$  (heat lost through the surface) are all rates of heat transfer. We need to quantify each of these.

Radioactive heating in the mantle is the mostly result of decay of  $^{238}\text{U}$ ,  $^{235}\text{U}$ ,  $^{232}\text{Th}$ , and  $^{40}\text{K}$ . In total these have a time-dependent heating rate of  $H_R$  in units of Joules per kilogram. We will not worry about the details of this heating rate, but will express the total rate of radioactive heating in terms of its ratio with the surface heat loss:

$$\text{Ur} = \frac{Q_R}{Q_S} = \frac{MH_R}{Q_S}, \quad (42)$$

where  $\text{Ur}$  is a nondimensional number called the Urey ratio.

The surface heat flux can be approximately modelled using the Nusselt number. Recall that  $\text{Nu} = q_{\text{total}}/q_{\text{conductive}} \propto \text{Ra}^{1/3}$ . Rearranging this and assuming the constant of proportionality is about unity gives

$$\begin{aligned} q_S = q_{\text{total}} &= q_{\text{conductive}} \text{Ra}^{1/3}, \\ &= \frac{kT}{h} \left( \frac{g\rho_0\alpha Th^3}{\kappa\mu(T)} \right)^{1/3}, \end{aligned} \quad (43)$$

where we have used the mean mantle temperature  $T$  as a proxy for  $\Delta T$ , since the surface temperature is constant. Temperature also appears implicitly in the viscosity  $\mu(T)$ . The rate of surface heat loss is then the heat flux times the area of the Earth's surface,

$$Q_S = 4\pi R_E^2 q_S, \quad (44)$$

and hence we can rewrite equation (41) as

$$\frac{dT}{dt} = \frac{1}{MC} \left[ (U_r - 1)4\pi R_E^2 kT \left( \frac{g\rho_0\alpha T}{\kappa\mu(T)} \right)^{1/3} + Q_C \right]. \quad (45)$$

We will not consider the core heat flow  $Q_C$  in detail; suffice to say that it can be reconstructed similar in a manner similar to the surface heat flow.

In the end, we have an ordinary differential equation for the mean mantle temperature,  $T$ . This equation can be solved numerically forward in time, given an initial condition, or it can be solved backward in time from the present state. Figure 9 shows the results of a slightly more complicated model with independent temperatures for the upper and lower mantle. It was solved forward in time from an initially hot Earth.

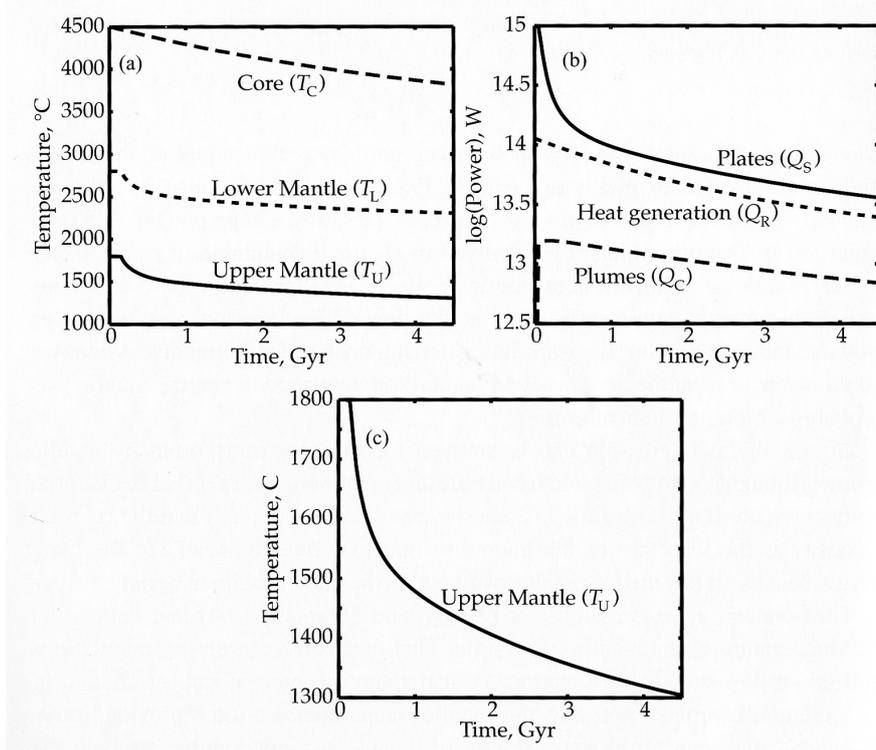


Figure 9: Results of a reference thermal evolution model of the mantle and core.

We can notice several things about Figure 9:

- There is a rapid cooling phase during the first half billion years of the model. This is the phase when the initial heat of the mantle is lost through the surface, and the temperature goes toward the temperature that can be maintained by radioactive heating.

- There is a slower cooling for the rest of the model time, where the surface heat flow  $Q_S$  is approximately parallel to the radiogenic heating  $Q_R$ . (If  $Q_R$  were constant,  $Q_S$  would approach it asymptotically).
- The heat flow from the core (by mantle plumes) remains very low through the entire run, which keeps the core temperature very high.
- The overall picture is that primordial heat is lost quickly and for most of the evolution of the model, the temperature is set by a balance between surface cooling and radiogenic heating.

At the end of this run, the surface heat flow is about 36.4 TW and the core heat flow is about 7.6 TW — both are close to the real present Earth value. The total radiogenic heating is about 24.6 TW at the end of the model, so the final Urey ratio is about  $24.6/36.4 \approx 0.7$ . And the heat flow corresponding to secular cooling is about  $36.4 - 7.6 - 24.6 = 4.2$  TW. Recall that

$$\frac{dT}{dt} = \frac{Q_{\text{net}}}{MC}, \quad (46)$$

and so with  $M = 4 \times 10^{24}$  kg and  $C = 1000$  J/kg/K, the predicted cooling rate is about 30 K per billion years.

So the model tells us that if the surface heat flow of the Earth is nearly balanced by radiogenic heating, the Urey ratio must be near 0.7. Geochemists, in contrast, tell us that the “observed” Urey ratio of the Earth is more like 0.3! This means that the present-day heat flow out of the mantle is not supported by radiogenic heating! There are at least three possible explanations

1. The geochemists are wrong, and there are more heat-producing elements in the mantle than they think.
2. The present rate of heat loss is anomalously high at the moment, representing a temporary fluctuation in mantle convection. If we averaged over several fluctuations (say a few hundred Ma), we would find that the heat flow is balanced by the observed radiogenic heating. In this scenario, the present cooling rate would be about 140 K per billion years. Half a billion years ago, the mantle would have been 70 K hotter, and there would have been a lot of magma around! We don’t see evidence of this. So the fluctuations in this scenario would have to be shorter term. This could come from something that has been called episodic convection.
3. The present rate of heat flow is high, but not anomalously high. The rate of cooling of the Earth is large, and the planet was much hotter in the past. Extrapolating back naively, this gives unrealistically high temperatures in the past (magma ocean 1 Ga ago). This is known as the “thermal catastrophe.” Of course there was no thermal catastrophe in the past! This just indicates that our understanding of mantle convection is flawed.

For case 3, there have been a number of possible scenarios advanced. One model from the recent literature<sup>1</sup> suggests that the mantle was molten at its base for much of the Earth's history. The heat flow that we see today that is unbalanced by radiogenic heating came not from secular cooling of the mantle, but secular freezing of the basal magma ocean! Other possibilities are discussed by Geoffrey Davies in his book.

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<sup>1</sup>Labrosse, Hernlund, and Coltice, A crystallizing dense magma ocean at the base of the Earth's mantle, *Nature*, 2007.