VEIN SPACING IN EXTENDING, LAYERED ROCK: THE EFFECT OF
SYNKINEMATIC CEMENTATION

JOHN N. HOOKER*† and RICHARD F. KATZ**

ABSTRACT. Cemented fractures (veins) commonly show mm-scale spacing, even in
layerbound arrangements in beds of cm- to dm-scale thickness. The relief of tension
around such layerbound fractures should preclude nearby fracture propagation and
result in cm- to dm-scale fracture spacing. We hypothesize that cement precipitated
during vein opening could re-establish tension across veins and lessen the effects of
relieved tension, thus decreasing fracture spacing. We test this hypothesis using a
computer-based numerical model. The model consists of a 2D triangular lattice of
nodes connected by elastic springs. The lattice is stretched by holding the left-
boundary stationary and moving the right-boundary to the right at constant velocity.
The lattice consists of three layers; springs within the middle fracturing layer
fail upon stretching past a given critical length. Springs within the upper and lower matrix layers
are indestructible. We tune the model parameters to produce the familiar regular
spacing of barren fractures (joints). Then we perturb this system by adding cement
within fractures as the fractures propagate and widen. Cementation is simulated by
extending failed springs across the space between the nodes on which they are rooted
and re-attaching springs once they reach across. Re-attached springs are assigned a new
neutral length equal to their current length on re-attachment. The primary effect of
cementation is to make fractures narrower and more closely spaced. Thus the model
highlights the resistance to fracture widening by cement as a potential reason why veins
can be more closely spaced than joints. Individual fractures open and seal multiple
times when the stiffness of cemented springs is lower than that of the host-rock
springs, suggesting that natural crack-seal vein opening is associated with persistent
mechanical weakness at extant fractures. Modeled veins are more irregularly spaced
than modeled joints, but the vein patterns remain more regularly spaced than would be
expected for a random arrangement. Therefore the model does not explain systematic
clustering of natural veins.

Key words: fracture spacing, fracture model, vein, crack-seal, diagenesis

INTRODUCTION

The spacing of opening-mode fractures that are bound within a sedimentary layer
is generally observed to be regular and proportional to layer thickness. This pattern
has been explained by a number of models that, while different in their details, all
invoke tension relief adjacent to extant fractures. This tension relief extends a lateral
distance proportional to fracture height and inhibits the propagation of fractures in
that zone (Price, 1966; Hobbs, 1967; Gross and others, 1995; Schöpfer and others,
2011). The result is a characteristic fracture spacing with a fracture-spacing:layer-
thickness ratio (S:T) near 1, but ranging from 0.1 to 10 in layers of cm- to dm-scale
thickness (Ladeira and Price, 1981; Bai and Pollard, 2000a and references therein).

In contrast, some natural fractures have been observed with relatively close or
irregular spacing (fig. 1). Departures from the expected fracture-spacing pattern have
been attributed to a variety of causes. Irregular spacing of fractures may emerge in the
absence of binding layers (Gillespie and others, 2001), by changes in layer thickness or
mechanical properties (Zahm and others, 2010), with proximity to folds or faults

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(Peacock, 2004), or as a result of subcritical fracture propagation (Olson, 1993, 2004), during which clusters of fractures can propagate simultaneously, before tension-relief shadows preclude nearby fracture growth. Anomalously close spacing (S:T < 0.1) has been noted in particularly thick beds (T > ~ 1 m) in sedimentary sequences whose thinner-bedded counterparts have a more typical S:T (Ladeira and Price, 1981).

Furthermore, the presence of cement in layerbound fractures is often associated with close or irregular spacing. In many cases, closely or irregularly spaced vein patterns can be attributed to one of the factors mentioned above. However, we show below, in a review of the literature, that there remain differences in fracture spacing between veins and joints, even when controlling for such external factors, where they can be identified. This observation compels the hypothesis that the defining characteristic of veins—cement—may affect fracture spacing to promote close or irregular spacing. Cementation of fractures and its consequences for fracture spacing are the focus of the present study. We shall use the terminology of Peacock (2004) and refer to fractures that contain cement as veins, whereas barren fractures we refer to as joints.

This manuscript is structured as follows. In the next section we review the literature of natural joint and vein patterns in order to establish systematic contrasts between the two that are not adequately explained. The following section contains a thorough mathematical description of the model, including details of the calibration and physical assumptions regarding cementation and fluid pressure. Then in the Results section we interpret the output of the model by analyzing fracture patterns formed under different cementation rates and mechanical properties. The key result

Fig. 1. Closely-spaced veins in outcrop (A), Blue Lias Formation, Somerset, UK; and in thin section (B), Utica Shale, Pennsylvania, USA. In (A) a fractured layer (FL) lies between two non-fractured layers (NFL). Both veins (V) and joints (J) generally terminate at the boundaries of the fractured layer. At the scale of the photograph, carbonate cemented veins have a much closer spacing than barren joints. At larger scales, veins are clustered around normal faults (Putz and Sanderson, 2008). In (B), the thin section is map-view and unoriented in azimuth. All veins are sealed with carbonate cement. An early vein (horizontal in map view) contains crack-seal (CS) texture, marking increments of fracture opening. Crosscutting veins form two clusters, resembling the crack-jump (CJ) texture described by Caputo and Hancock (1999). It is unclear whether the crack-jump mechanism (Caputo and Hancock, 1999) formed this vein clustering, because it is unknown whether these microscopic veins are layerbound.
we obtain is that with increasing cementation rate, fractures become narrower and more closely spaced. In the Discussion we interpret the behavior of the model. We conclude that although the model does not account for some aspects of vein size- and spacing-patterns observed in nature, the effect of fracture cementation does result in closer and more irregular vein spacing. Thus we conclude that resistance to fracture widening is a primary mechanical effect of synkinematic (during-opening) vein cementation in natural rocks.

**Previous Work**

The layerbound vein populations summarized in table 1 illustrate the relatively close and irregular spacing of veins versus joints. Regularity of spacing is quantified here using the coefficient of variation or dispersion ($C_v$), that is, the standard deviation of spacings divided by the mean. The $C_v$ will be near 1 for a random arrangement, less than 1 for regular spacing, and greater than 1 for systematically clustered arrangements (for example, Gillespie and others, 1999).

Thin sections from vein-hosting sedimentary rocks, including many entries in table 1, commonly include closely and irregularly spaced, microscopic veins (for example, Nemčok and others, 1995; Laubach, 1997; Guerriero and others, 2011). As such, many vein populations are top-bound (Hooker and others, 2013), in that only the largest veins present are long enough to have reached the layer boundaries and so be height-restricted. Many joint populations are also likely top-bound, but the existence of microscopic joints is difficult to distinguish from artificial fracturing in thin-section. The data in table 1 were collected at the outcrop scale and thus are dominated by bedding-bound fractures, except as noted.

In figure 2 we illustrate the greater range of spacing irregularity and closer average spacing of veins relative to joints. This figure omits spacing data from vein and joint populations whose intensity varies systematically around folds, faults, layer-thickness changes, or other external clustering controls (table 1). In doing so, figure 2 highlights the differences between joint and vein spacing that are not accounted for by external factors. For example, although the veins in figure 1A are layerbound, these veins are clustered at the tens-of-meters scale near normal faults, which cut multiple layers (Putz and Sanderson, 2008). The resulting high $C_v$ of this population (table 1) reflects the larger-scale distribution of the normal faults, while the cm-scale spacing remains unexplained.

Crack-seal texture (Ramsay, 1980) is common in veins and demonstrates that individual veins do not open in a single step but in multiple brittle-fracture events. These events are marked in the vein by fracture-parallel bands of cement, or crack-seal increments. During the opening of such veins, tension across the vein is sufficient to repeatedly break the cement that fills the vein, or the bond between the host-rock and the cement (Cox, 1987; Hilgers and Urai, 2002; Laubach and others, 2004).

Caputo and Hancock (1999) suggested that cementation could re-seal fractures and allow tension to re-build across them. That study described crack-jump as a semi-localized variation of crack-seal, in which successive crack-opening and sealing increments form near previous fractures, but leave some finite distance of host-rock in between. Thus that study identified a potential process for close vein spacing.

Both crack-seal and crack-jump textures imply a fracturing process in which tension was restored across the fracture during opening. However, the mechanism that restores tension is not obvious. Cement precipitation in extension veins could restore tractions across previously tractionless barren fractures, but the cement would presumably be relaxed upon sealing. Some veins are thought to open by force of crystallization (Means and Li, 2001; Wiltschko and Morse, 2001). In such cases, chemical potentials cause the precipitation of vein cements at the cement—host-rock boundary and actually force the fracture walls apart, such that the cements grow antitaxially. But
## Table 1

**Measured joint and vein patterns from the literature**

<table>
<thead>
<tr>
<th>Author</th>
<th>Year</th>
<th>Fractures cemented?</th>
<th>Fractures layer-bound?</th>
<th>Spacing distribution type</th>
<th>C, Externally imposed clustering?</th>
<th>S/T ratio</th>
</tr>
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<tr>
<td>McQuillan</td>
<td>1973</td>
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<td></td>
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</tr>
<tr>
<td>Narr and Suppe</td>
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<td>No</td>
<td>Yes</td>
<td>N</td>
<td>No</td>
<td>0.8</td>
</tr>
<tr>
<td>Rivers and others</td>
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<td>Yes</td>
<td>LN, E</td>
<td>No</td>
<td>0.9—1</td>
</tr>
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<td>Gillespie and others</td>
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<td>Yes</td>
<td>N</td>
<td>No</td>
<td>0.9</td>
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<td>Becker and Gross</td>
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</tr>
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<td>Gross and others</td>
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<td>Ruf and others</td>
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<td>Yes</td>
<td>LN</td>
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<tr>
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<tr>
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<td>Yes</td>
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<td>Yes</td>
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</tr>
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<td>Cooke and others</td>
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<td>No</td>
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</tr>
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<td>Larson and others</td>
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<tr>
<td>Gross and Engelder</td>
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<tr>
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<td>Hooker and others</td>
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<td>Gillespie and others</td>
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<td>No</td>
<td>P</td>
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<td>Yes</td>
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<td>Gross and others</td>
<td>1995</td>
<td>Yes</td>
<td>Yes</td>
<td></td>
<td>0.4—0.9</td>
<td>Yes</td>
</tr>
</tbody>
</table>

Blank: neither reported nor inferable; ?: inferred, not reported; .: calculated or approximated from data reported but not explicitly reported, or reported as approximate.

Externally imposed clustering means clustering around geological features such as folds, faults, dolomitized zones, and changes in layer thickness.

S/T ratio based on mean spacing; see text.

E: exponential; G: gamma law; LN: lognormal; N: normal; P: power law; PS: positively skewed.

NA: not applicable.

Multiple entries from the same reference correspond to different fracture populations.

**Notes**

1. S/T lower for thick layers.
2. Constant S with increasing T above some value, S/T ≥ 1 for thickest layers.
3. S/T is the reciprocal of 1.3, their fracture spacing index.
4. Nash Point joints measured far from faults, which do modify joint spacing.
5. Only unsystematic joints clustered externally.
7. Joints contain postkinematic cement.
8. Spacing changes around fault zones but C, reported for zones of constant spacing.
9. S/T of 0.4 in one layer to accommodate fault slip. Otherwise no consistent spacing change around fault.
10. Layer-bound fractures have higher kurtosis than ideal lognormal for non-layer-bound fractures.
11. Some populations have positive skew from normal.
12. Study avoids closely spaced fractures near faults.
13. One C, outlier ~1.5.
14. Two positive S/T outliers (~4 and ~6.67).
15. Beleite outcrop.
16. Spacings lognormal in aggregate, interpreted as trilithic. Clustering attributed to hydrofracture propagation.
17. Set 3: joints contain postkinematic cement.
18. Cement in 27% of fractures.
19. Layer-bound fractures analyzed separately.
20. 10% positive S/T outliers, max 6.5.
22. Clustering attributed to non-saturation.
23. Top-bound (microfractures not layerbound).
24. Lognormal distribution for layerbound fractures.
25. Data from Chapter 7 other than Tranquitas dataset.
26. Kimmeridge data from vein lines taken far from thicker veins near faults.
27. Kilve data.
28. Eifel area.
29. Ardennes area.
30. Sample I macrofractures. Other samples have various or unknown layer-boundedness.
31. Tranquitas dataset.
32. Set 1.
tectonically deformed environments commonly include veins with syntaxial or “stretched” (Bons and others, 2012) crystal textures as well as true crack-seal textures, which demonstrate that the vein material itself was repeatedly broken and displaced as the vein grew (Bons and others, 2012; Hooker and others, 2015). If fractures act as valves (Sibson, 1992), then cement might reduce permeability, raising fluid pressure and thus promoting further fracturing. However, this seems an unsatisfactory explanation for isolated, opening-mode fractures. Valve action (Sibson, 1992) and general fluid-pressure cycling related to fracture cementation (Boullier and Robert, 1992; Rusk and Reed, 2002) are primarily attributed to faults and interconnected vein networks that may cut multiple stratigraphic intervals and form interconnected pathways for fluid flow.

More recent empirical work has further highlighted the mechanical effects of cement phases during brittle deformation. Nüchter and Stöckhert (2007) used electron backscatter diffraction to detect dislocations within quartz cement in stretched veins, showing that such dislocations were concentrated in older quartz cements. Thus that study highlighted a case in which vein deformation proceeded during vein cementation. Silica redistribution during fracturing was postulated as a mechanism to restore tensile strength on fault slip interfaces (Fisher and Brantley, 2014).

Numerical modeling work has followed a parallel track in demonstrating the importance of diagenesis on the mechanics of fracture opening. A common approach to modeling layerbound extension fractures is to use a lattice of springs, which may break upon stretching past a critical distance (for example, Koehn and others, 2005).
Spence and Finch (2014) used such an approach to show that mechanical contrasts in diagenetically altered layers (for example, chert beds in a limestone sequence) can affect fracture spacing and size distributions. Virgo and others (2013) simulated re-cracking of a vein using a 3D spring-lattice model. That study suggested that whether a new crack forms along or nearby an extant vein depends on the relative strengths of the vein material and the host rock, as well as the angle between the extant vein and the new extension direction. The same authors later used a similar approach to model the opening of sets of veins after rotating the stress field, reproducing crack-seal and crack-jump behavior (Virgo and others, 2014). Vass and others (2014) took a critical step of incorporating cementation, as well as fluid pressure changes, concomitant with the layer extension that produces the original fracture pattern. This cementation was achieved by re-attaching broken springs after some time, according to a prescribed cementation rate. That study suggested that fracture patterns and fluid-flow in overpressured environments are sensitive to diagenesis. Here we use a similar yet simplified approach to isolate and model the effect of cement precipitation on the classic problem of extension-fracture spacing in layered rock.

**Spring-lattice model**

We wish to address the physical problem of cementation of veins that develop in extending rock. We model extending rock as a lattice of linear springs connected at a regular array of nodes. These springs are allowed to break when they reach a specified amount of extension. Ostoja-Starzewski (2002) reviews the literature on spring-lattice network models, including the variants of lattice topology (for example, triangular, square, random) and spring forces (normal force, shear force, and bending moment). Our interest is in a layer under extension, and hence the relevant forces are dominantly tensile. We therefore follow Curtin and Scher (1990a, 1990b) by adopting a triangular lattice but neglecting the shear forces and bending moments of the springs. The details of the formulation are explained below. Curtin and Scher (1990b) apply this model to a range of problems from traditional continuum and fracture mechanics and find excellent agreement in the predicted behavior, as long as key assumptions about the resolution of features hold.

Although there are rigorous, asymptotic equivalences between spring-lattice and continuum fracture-mechanics approaches (Ostoja-Starzewski, 2002), the discrete nature of spring-lattice models complicates their interpretation in the geological context. A key challenge is that the elements of a spring lattice—the springs and nodes where they connect—have no exact correspondence to a natural rock bed. Therefore any confidence in the capability of such a model to simulate the effect of cementation is principally derived from the model’s ability to quantitatively reproduce the expected rock-behavior in the absence of cementation. This motivates a calibration of the model parameters to reproduce the regularly spaced, layerbound joint pattern under zero cementation. The effect of cementation is then considered as a perturbation to this familiar system.

**Model Overview**

Here we briefly outline the numerical model. The interested reader can find a rigorous mathematical description throughout the rest of this section, but may otherwise skip to the Results.

The present model consists of a lattice of springs connected by nodes in a triangular pattern (fig. 3). Deformation is purely linear-elastic. The domain includes three horizontal layers. Fractures form in the middle layer (fracturing layer) only. Each spring in the middle layer is assigned a spring constant and breaking length; once any spring extends beyond its breaking length it is considered to have broken. Springs in the upper and lower layers (matrix layers) have a spring constant which is uniform but...
independent from that of the fracturing layer. Springs in the matrix layers are indestructible and so have no breaking length. A boundary condition extends the right-hand boundary toward the right at a fixed rate, thus stretching the lattice.

We incorporate cement precipitation into our model by lengthening and reattaching broken springs. Broken springs grow at a constant rate; spring reattachment occurs when two segments of a growing spring span the distance between the nodes where the springs are rooted. Neither the rate of extension nor cementation vary throughout the simulations.

In nature, subsurface fracture opening and cementation both occur in the presence of fluids. Fluids are thought to decrease the effective stress such that opening-mode fractures might form instead of faults (Secor, 1965). Moreover, the cementing phase is transported by fluids that circulate within the rock and precipitates to satisfy thermodynamic constraints (Bjørlykke, 2014). While previous work has incorporated fluid pressure into lattice models (Vass and others, 2014), we choose not to explicitly model fluid pressure here, for three reasons. First, to do so is computationally expensive; omitting fluid pressure allows us to increase the resolution of our lattice. Second, neglecting fluid flow in this model is compatible with the assumption that bedding extension drives fracturing. Fluid pressure is sufficient to reduce the overburden and allow fracture opening, but local fluid-pressure variations have a negligible effect on fracture patterns. Third, as stated in the Introduction, the strength

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**Fig. 3.** The domain and lattice at the initial time. (A) Schematic drawing of a patch of the spring lattice before any deformation has occurred. (B) Schematic drawing of the domain, showing the fracturing layer bounded by the unbreakable matrix layers. Homogeneous strain is enforced along the top and bottom row of nodes (dashed lines).
of our modeling approach lies in our ability to reproduce joint spacing in extending rock as commonly observed and then to see how that pattern changes as we vary the cementation rate. Although fluid pressure variations in time and space could certainly affect vein patterns, such variations could also affect joint spacing, and the classic joint-spacing models do not account for fluid-pressure fluctuations. By restricting our attention to changes in cement precipitation rates and mechanical properties, we focus on how cement modifies the accepted model of joint-pattern development.

Likewise, our elastic-lattice approach neglects inelastic deformation. Inelastic processes such as pressure-solution creep likely play an important role in natural veining, as evidenced by similar isotopic compositions between host-rock carbonates and calcite vein cements (Dietrich and others, 1983; Evans and others, 2012), suggesting the vein fill comes from local host-rock dissolution. Such a process could be incorporated into an elastic lattice model by increasing the neutral length of a stretched spring to a value closer to that of its stretched length, or by balancing springs lengthened by cementation with other springs shortened by dissolution. However, the goal of the present study is to modify the classical joint-spacing models, which assume linear-elastic host-rock behavior, by incorporating cementation alone. In doing so, we take the most parsimonious approach we can in order to explain the contribution of cementation to differences between joint and vein spacing.

The Spring Lattice

We use a two-dimensional, triangular lattice of springs with initial length $\Delta_0$, as shown in figure 3A. Nodes are indexed in a logically rectangular, two-dimensional array with rows $j \in [0, N_J \cdot 1]$ and columns $i \in [0, N_I \cdot 1]$. At alternate rows ($j = 1, 3, 5 \ldots$), the initial $x$ position of the nodes for all $i$ are offset in the $x$ direction. More specifically, the initial condition is

$$x_{ij}(t = 0) = \Delta_0 \left[ \sqrt{3} \left( i + \frac{j \mod 2}{2} \right) \hat{x} + \frac{j}{2} \hat{y} \right],$$

where $\hat{x}$ and $\hat{y}$ are unit vectors in the $x$ and $y$ directions, respectively. The array of nodes has dimensions of $N_i \times N_j$, and for symmetry we require $N_j$ to be an odd number. This leads to a layer of height $H = \Delta_0 (N_J - 1)/2$ and length $L = \Delta_0 (N_I - 1) \sqrt{3}$. The assumption that is inherent in the use of a 2D model is that there is no variation in the third dimension: that all in-plane features extend uniformly in the out-of-plane direction. This assumption is generally made in 2D numerical models of height-restricted fractures, on the basis that layer-parallel fracture length is much greater than fracture height. Likewise, this assumption is preferable to taking on the computational expense of modeling the third dimension when the focus of study is fracture spacing, which is typically measured in 1D.

The boundary conditions forcing the layer to extend are specified as

left: $x_{0j} \cdot \hat{x} = 0$ (for $j$ even),

right: $x_{(N_J-1)j} \cdot \hat{x} = \sqrt{3} \Delta_0 \left( N_J - 1 + \frac{j \mod 2}{2} \right) + Vt$ (for all $j$),

where $V$ is the rate of displacement of the right-hand boundary and $t$ is the elapsed time. To keep the layer oriented along the $x$-axis, we also enforce no displacement in the $y$ direction for one node at the left and right boundaries:

left: $x_{0(N_I-1)/2} \cdot \hat{y} = 0$,

right: $x_{(N_I-1)(N_J-1)/2} \cdot \hat{y} = 0$. 

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John N. Hooker and Richard F. Katz—Vein spacing in extending,
The $x$ position of nodes along the top and bottom of the domain can also be forced to satisfy a boundary condition enforcing uniform extension,

$$\text{bottom: } x_{i0} \cdot \hat{x} = x_{i0} (t = 0) + \frac{i}{N_i - 1} Vt, \quad (4A)$$

$$\text{top: } x_{iN_i-1} \cdot \hat{x} = x_{iN_i-1} (t = 0) + \frac{i}{N_i - 1} Vt. \quad (4B)$$

All $x$ and $y$ positions that are not specified in equations (2), (3), or (4) are computed to satisfy force balance.

**Balance of Forces**

At some time $t > 0$ when the lattice is under tension, the $(x,y)$ position of each node is determined by requiring that static forces balance in the $x$ and $y$ directions. Neglecting inertial forces and bending moments of springs, we have, for each node in the lattice,

$$\sum_{s=0}^{5} F_{ij}^{(s)} = 0 \quad (5)$$

where $F_{ij}^{(s)}$ is the force vector due to spring $s$ on node $ij$. We sum over the six springs connected to the node, with each spring force given by

$$F_{ij}^{(s)} = K_{ij}^{(s)}(|\Delta_{ij}^{(s)}| - \hat{\Delta}_{ij}^{(s)})\hat{\Delta}_{ij}^{(s)}, \quad (6)$$

where $K_{ij}^{(s)}$ is the spring constant, $\Delta_{ij}^{(s)}$ is the vector originating at node $ij$ and ending at the node connected by spring $s$, $\hat{\Delta}_{ij}^{(s)}$ is the neutral length for that spring, and $\hat{\Delta}_{ij}^{(s)} = \Delta_{ij}^{(s)}/|\Delta_{ij}^{(s)}|$ is a unit vector pointing in the direction of $\Delta_{ij}^{(s)}$. Initialization of spring constants over the lattice is discussed below (Material Heterogeneity and Layering).

Equation (6) states that the magnitude of the force is proportional to the amount of extension beyond the spring’s neutral length; the force is directed exactly along the spring. As an example, consider spring $s = 0$ at node $ij$, where $j$ is even. There we have

$$\Delta_{ij}^{(0)} = x_{ij+1} - x_{ij}. \quad (7)$$

The indexing for springs $s = \{0, 1, 2, 3, 4, 5\}$ is different for $j$ even and $j$ odd but can be determined from figure 2A. This configuration of springs and forces gives rise to an effective Poisson’s ratio of $1/3$ for the unfractured lattice (Ostoja-Starzewski, 2002).

**Non-Dimensionalization and Assembly**

It is convenient to rescale the variables to make them dimensionless and of order one. To do so, lengths are scaled with $\Delta_0$ and time is scaled with $\Delta_0/V$. Equation (1) for the initial positions of nodes before any extension is applied becomes

$$x_{ij}(t = 0) = \left[\sqrt{3}\left(i + \frac{j \mod 2}{2}\right)\hat{x} + \frac{j}{2}\hat{y}\right], \quad (8)$$

and the boundary condition (eq 2B) becomes

$$x_{i(N_i-1)j} \cdot \hat{x} = \sqrt{3}\left(N_i - 1 + \frac{j \mod 2}{2}\right) + t. \quad (9)$$
where all symbols are now rescaled to be dimensionless. Furthermore, we rescale the spring constant by a reference value $K_0$ and rescale forces by $K_0/\Delta_0$.

We can assemble all of the dimensionless force-balance constraints into a system of algebraic equations for the grand vector $\mathbf{X}$ containing all node positions, which are the $N_i \times N_j \times 2$ unknowns. This system has the form

$$G(\mathbf{X}) = 0,$$

where $G$ is a nonlinear algebraic operator. The non-linearity in equation (6) arises because the directions of the spring can change as node positions equilibrate under the force-balance constraint. We solve for an approximate numerical solution $\tilde{\mathbf{X}}$ using Newton’s method; Gaussian elimination determines the correction $\delta \mathbf{X}$ at each Newton iteration. We accept a numerical solution and terminate the Newton iterations when the L2 norm of the nonlinear residual is sufficiently small:

$$\|G(\tilde{\mathbf{X}})\|_2 \leq \text{tol.},$$

where the tolerance is set to be $10^{-6}$. In rare cases for time-steps containing large fracture events, Newton’s method takes more than 20 iterations and the residual stalls at $\sim 10^{-4}$. Visual inspection of maps of the residual $G$ indicates that this stalling results from a few isolated nodes at the margin of a new fracture. To overcome this stalling, we begin at iteration 20 to measure the size of the solution correction, $\|\delta \mathbf{X}\|_2/\|\mathbf{X}\|_2$. If this quantity is below $10^{-5}$ we accept the solution as converged. All calculations are performed in the software context of the Portable Extensible Toolkit for Scientific Computation (PETSc—Balay and others, 2001, 2004), extended with the MUMPS solver package (Amestoy and others, 2001).

### Time-Stepping and Spring Breakage

To compute the time-evolution of the node positions, we discretize model time into steps of size $\Delta t$. For each time-step, the displacement of the right-hand boundary is obtained using equation (2B); we use the force-balance given in equation (5) to solve for the remaining unknowns.

Once an acceptable solution has been obtained, all the springs are checked against a breakage criterion:

$$\text{if } |\Delta_{ij}^{(s)}| \geq \tilde{\Delta}_{ij}^{(s)}, \text{ then set } K_{ij}^{(s)} = \epsilon_K,$$

where $\tilde{\Delta}_{ij}^{(s)}$ is the breaking length for spring $s$ from node $ij$. Initialization of breaking lengths throughout the lattice is discussed below (Material Heterogeneity and Layering).

Equation (12) states that once a spring’s length has exceeded the pre-specified breaking length, it is considered to have broken, and its spring constant is set to a very small (but non-zero) number $\epsilon_K$. This small spring constant is typically four orders of magnitude less than the average spring constant for non-broken springs. This non-zero value is large enough to ensure that the Jacobian matrix at each Newton step is full rank, but small enough that the spring has a negligible effect on the dynamics.

If a force-balance solution leads to extension of one or more springs past its breaking length, then the solution is recomputed at the same time $t$ with either (i) $K$ for all broken springs updated according to equation (12) or (ii) $K$ for only the single spring stretched farthest past its breaking length updated according to equation (12). Option (ii) is likely more realistic in that it accounts for re-equilibration after the first spring breaks (for example, Koehn and others, 2005), but this option slows down our model considerably. We found no statistically significant change in fracture spacing resulting from option (i) versus option (ii), so we used option (i) for our investigations.
In either case, the new force-balance solution may cause more springs to extend past their breaking length. The force-balance/breakage procedure is iterated until no more springs break. Model time is then augmented by $\Delta t$ to begin a new time-step.

The time-step size $\Delta t$ can vary between steps. If Newton’s method at a time-step fails to converge, $\Delta t$ is reduced by a factor of $1/4$ and the solver is restarted. After a successful time-step during which no springs break, $\Delta t$ is increased by a factor of 1.1. If $N_b$ springs break during a successful time-step, $\Delta t$ is reduced by a factor of $(1 + 0.1 N_b)^{-1}$. The time-step size is limited such that $\Delta t \leq 1$; this means that in dimensional terms, the right-hand boundary is displaced by no more than $\Delta_0$ over one time-step.

Monette and Anderson (1994) employed this approach and found that for a lattice of uniform springs under tension, if a single spring is clipped, the resulting fracture will grow by breaking springs along lattice diagonals, whereas the geological expectation is for joints to propagate normal to the direction of maximum tension. This problem is indicative of the anisotropy inherent in a lattice. One option to drive straighter fractures would be to use a square lattice, but we thought it important to use a configuration that imparts a bulk Poissonian effect to the layer, because such effects are thought to be important in controlling joint spacing (Bai and others, 2000). Monette and Anderson (1994) partially remedied the problem of lattice-diagonal propagation by incorporating the bending moment of springs into the force balance. We have found that incorporating heterogeneity in the spring constant and breaking length, as discussed in the next section, enables the lattice to simulate regular joint patterns in the absence of cement.

**Material Heterogeneity and Layering**

Natural rocks are mechanically heterogeneous on a wide range of scales, certainly including the grain scale. Previous studies have modeled this heterogeneity by imposing randomly distributed small cracks (Olson, 1993) or by stochastically modifying material properties of lattice elements (Tang, 1997). We experimented with both approaches and found the latter more successful in overcoming the tendency for fractures to grow along lattice diagonals. We allow for pseudo-random variation in the spring constant (both initial and cemented) and the breaking length. Our implementation follows Tang (1997) and Tang and others (2008), who achieved realistic brittle-failure behavior using a Weibull distribution of modeled element mechanical properties.

The Weibull distribution is given by the probability density function

$$p(u) = \frac{m}{u_0} \left( \frac{u}{u_0} \right)^{m-1} \exp\left[ -\left( \frac{u}{u_0} \right)^m \right],$$

(13)

where $u > 0$ is the randomly varying property and $u_0$ and $m$ are scaling and shape parameters, respectively. At all values of $m > 1$, there is a peak in the distribution that tapers to zero at $u = 0$ and $u \to \infty$ (fig. 4). For increasing values of $m$, the distribution has a taller, narrower peak in probability density near $u/u_0 = 1$.

The dimensionless breaking length of each spring is initially set as

$$\tilde{\Delta}_ij^{(i)} = 1 + B_0 + B_p u^* (m),$$

(14)

where $B_0$ is a reference breaking length, $B_p$ is an amplitude factor for the breaking-length perturbation, and $v^* (m)$ is a pseudo-random number drawn from a Weibull distribution with shape parameter $m$. We also include a perturbation to the spring constant. In calibrating the spring-lattice model to produce a joint pattern in the limit of zero cementation, we obtained our best results using a uniform distribution of pseudo-random numbers $v \in [-1,1]$ and
where $K_p < 1$ is a dimensionless amplitude factor for the spring-constant perturbation. The neutral length of springs is initialized to unity (or, dimensionally, to $\Delta_0$) without perturbation. Other parameter values are given in table 2.

Thus the lattice springs are heterogeneous in their breaking lengths and spring constants. The corresponding material properties we wish to simulate and vary, both at the layer- and grain-scale, are tensile strength and stiffness (Young’s modulus). In the Discussion we address how these variations might correspond to natural rock features.

In natural rock beds, the mean characteristics of the granular-scale mechanical heterogeneity might vary at a larger spatial scale; for example, in sandstone beds that are graded or in mudrock beds that contain nodules or considerable fissility from large mica grains (for example, Arslan and others, 2012). For present purposes we ignore such complexities and generate perturbations $u^*$ and $v^*$ with independent random numbers. This means that $K$ and $\Delta$ are spatially random and uncorrelated.

Our present interest is on fracture patterns that develop in a single layer under extension. The behavior of such a layer, which we term the fracturing layer, is controlled, in part, by the mechanical properties of the adjacent layers (matrix layers), and the mechanical coupling at their interfaces. Here we limit our attention to an idealized case: the matrix layers are continuations of the fracturing-layer spring lattice, but are formed of homogeneous springs that cannot break (fig. 3B). The spring

$$K^{(s)}_{ij} = 1 + K_p v^*, \quad (15)$$

Fig. 4. The Weibull distribution for three values of $m$. 

$u_{0}p(u)$
constants of matrix springs $K_m$ are set independently of those of the fracturing layer, so the matrix layers can be either stiffer or more compliant than the fracturing layer. The matrix layers serve to isolate the fracturing layer from the lattice edges, and thereby to impose a more natural boundary condition than the homogeneous extension enforced at the top and bottom of the domain (eq. 4). The forces in the fracturing layer, particularly at the ends of fractures, are transmitted into the matrix layers and affect the elastic deformation there.

Cementation

Cementation of fractures by mineral precipitation can potentially promote fracture healing. The details of cement precipitation—including the material source, transport mechanism, and accumulation rates—are various and the subject of much previous work. Empirical study of cement volumes and morphologies in sandstones (Walderhaug, 1996) as well as diagenetic numerical modeling based on sandstones (Lander and others, 2008; Lander and Laubach, 2015) suggests that subsurface cement precipitation is commonly limited by kinetics, with accumulation rates dependent on temperature. Therefore, in many cases it is not excessively simplistic to neglect cement sources and model cementation by prescribing a uniform rate at which any spring can “re-grow” between two previously connected nodes that were disconnected by spring breakage. In the present model, cement grows equally from the two nodes toward each other at a dimensional speed $r_C$. We can formulate the following non-dimensional cementation criterion:

$$\text{if } 2R(t - \bar{t}_{ij}^s) \geq |\Delta_{ij}^s|, \text{ then reset } (K_{ij}^s, \tilde{\Delta}_{ij}^s) \text{ and set } \tilde{\Delta}_{ij}^s = |\Delta_{ij}^s|, (16)$$

where $\bar{t}_{ij}^s$ is the dimensionless model time when spring $s$ of node $ij$ broke. $R = r_C/V$ is a dimensionless parameter that controls the rate of cementation versus extension. The breaking time is recorded each time a spring breaks; then, after each time-step, the criterion (eq. 16) is applied to each broken spring. The neutral length $\bar{\Delta}_{ij}^s$ of a cemented spring is assumed to be equal to its length at the time of cementation. The dimensionless spring constant and breaking length are reset according to

$$K_{ij}^s = K_C + K_p \nu^s, \quad (17)$$
$$\tilde{\Delta}_{ij}^s = |\Delta_{ij}^s| + B_p \nu^s(m). \quad (18)$$

Table 2

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Ref. value</th>
<th>Range</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\varepsilon_K$</td>
<td>10$^{-4}$</td>
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<td>Spring constant for broken springs</td>
</tr>
<tr>
<td>$K_r$</td>
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<td></td>
<td>Mean spring constant for host-rock springs</td>
</tr>
<tr>
<td>$K_p$</td>
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<td></td>
<td>Amplitude of spring-constant perturbation</td>
</tr>
<tr>
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<td>0.5–5</td>
<td>Mean spring constant for cemented springs</td>
</tr>
<tr>
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<td>1</td>
<td></td>
<td>Spring constant for matrix-layer springs</td>
</tr>
<tr>
<td>$m$</td>
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<td></td>
<td>Shape factor in Weibull distribution of breaking lengths ($\Delta$)</td>
</tr>
<tr>
<td>$u_0$</td>
<td>0.1</td>
<td></td>
<td>Scale factor in Weibull distribution</td>
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<tr>
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<td>0–10</td>
<td>Cementation-rate factor</td>
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<td></td>
<td>Number of nodes in $i$-direction</td>
</tr>
<tr>
<td>$N_j$</td>
<td>201</td>
<td></td>
<td>Number of nodes in $j$-direction including matrix layers</td>
</tr>
<tr>
<td>$N_b$</td>
<td>30</td>
<td></td>
<td>Number of nodes in $j$-direction in each matrix layer</td>
</tr>
</tbody>
</table>
$K_C$ is the dimensionless ratio of the mean spring constant of a healed spring to $K_0$. In natural systems, cement can be more or less stiff than the host rock, depending on the mineralogy and lithology, corresponding to $K_C > 1$ or $K_C < 1$, respectively.

In considering the physical meaning of $R$ (eq. 16) we encounter a subtlety in the interpretation of the model: at fixed values of $r_C$ and $V$, the importance of cementation will increase with grid parameter $N_i$. This is because the strain rate in the $x$ direction at $t = 0$ is $\dot{\varepsilon}_{xx} = V/(\sqrt{3}\Delta t N_i)$; more springs in $x$ means a longer layer, and hence a smaller strain rate, relative to the cementation rate. To correct this and ensure that the cementation rate has a clear physical meaning, we force $r_C$ to vary in proportion to the rate of extension of a segment with length equal to the layer height $H$. This choice is motivated by the fact that in the absence of cementation, the fracture spacing is approximately equal to $H$ (fig. 2). We choose to let

$$r_C = f\dot{\varepsilon}_{xx}H = f\left(\frac{V}{L}\right)H = fV \frac{(N_j - 1)/2}{\sqrt{3}(N_j - 1)}, \quad (19)$$

where $f$ is the parameter that we use to control $R$. With this formulation, $f$ is interpreted as approximating the ratio of cementation rate to the cementation-free fracture opening rate. We expect to see a transition to cement-dominated behavior in the range of $0 < f \leq 1$.

The cementation rate, parameterized by $f$, is applied equally across the domain. By doing so we obviously neglect complexities such as proximity to the source of the cement and variation in the saturation state of the precipitating fluid. Although these assumptions are not uniformly valid, they are consistent with interpretations of fracture cement textures observed in a wide range of tectonic settings, including that cementation partially or fully keeps pace with fracture opening within numerous isolated, parallel cracks (Laubach, 2003; Bons and others, 2012). We address potential consequences of our assumptions on fracture patterns in the Discussion.

Results

We investigate the effects of cementation of fractures during layer extension by varying $f$ while holding all other model parameters constant (table 2). For the first round of simulations we impose an identical initial heterogeneity-distribution to the spring lattice; this step allows us to eliminate the initial pattern as a source of variation in the results. Two subsequent rounds are identical to the first except that the pseudo-random assignment of heterogeneity is re-seeded for each simulation. These later rounds improve our statistics on fracture size and spacing.

Apart from the rate of cementation, parameterized by $f$, we also considered variation of the spring constant of cemented springs (table 2). We considered this variation in order to investigate rock-fracture systems in which the mechanical properties of veins contrast those of the host rock, whether by contrasting stiffness between the cement and host rock, by weakness of the bonds between cement and host-rock, or by incomplete sealing.

Before we present the results associated with non-zero cementation we focus on the reference (cement-free) case in more detail. This reference case allows us to examine how springs break, and thus how fractures form in the model. Specifically, we note a transition from diffuse spring failures to coherent, discrete fractures of the lattice.

Lattice Extension and Spring Breakage

Figure 5 shows that spring failure in the model begins as isolated events at relatively homogeneously distributed locations. This early, diffuse phase of spring breaking is a consequence of the pseudo-random allocation of mechanical heterogene-
ity; particularly weak springs are randomly distributed in space and break early. When any spring breaks, the tension it supported is transferred onto the surrounding springs. This transmitted tension is superimposed on the background tension, and at some point the two become sufficient to trigger cascades of failure. Thus begins a second, discrete phase of spring breaks. We address parallels between this pattern of spring breakage and natural fracturing in elastic material in the Discussion.

Cascades occur because tension transmitted from one broken spring imparts sufficient tension onto neighboring springs to break them as well, which transmits even more tension to further neighboring springs. Monette and Anderson (1994) noted that in the triangular lattice we use, cascades will preferentially move along lattice diagonals. Sufficiently high heterogeneity in spring breaking length and spring constant (table 2), which are spatially random, dominates over the anisotropy of the lattice and generally causes failure cascades to propagate perpendicular to the applied

Fig. 5. Pattern of breaking springs. Matrix-layer springs light gray; unbroken springs black; broken springs red; healed springs blue. (A) Lattice during early, diffuse phase of spring breakage. (B) Lattice soon after the onset of the late, discrete phase. (C) Lattice at final extension. Areas shown in detail from (A) illustrate the identical mechanical heterogeneity in the lattice; despite this, the final fracture pattern is different.
tension, as would be expected for fractures in isotropic material. Vertical shortening of
the lattice during lateral extension, visible in figure 5, suggests that the effect of
non-zero Poisson’s ratio is preserved.

Cascades of broken springs form structures in the lattice that we interpret as
fractures. Through calibration we modified the initial heterogeneity such that frac-
tures form as chains of broken springs that extend vertically across the fracturing layer,
albeit with some tortuosity (fig. 5B). This tortuosity appears to result from the high
heterogeneity. At high strains, further lattice damage tends to accumulate near the top
and bottom of the fracturing layer. This damage either takes the form of lateral
propagation of extant layerbound fractures or of short vertical fractures in between
layerbound fractures (fig. 5C).

The model parameters in equation (14) are calibrated so that fractures propagate
in a direction not dominated by the lattice diagonals. This calibration resulted in the
emergence of fractures at extensional strain $\varepsilon_{xx} \sim 0.05$ (fig. 5B). We produced fracture
patterns by extending our spring lattice to a final $\varepsilon_{xx} = 0.144$. In contrast, natural
fracture-sets generally record much lower strain (Hooker and others, 2009). By
reducing the breaking length, we could trigger fractures at smaller, more realistic
strains, but this would also make the fractures thinner. Wide fractures are more readily
distinguished algorithmically from diffuse spring breakage, so the minimum analyz-
able width of the fractures is effectively limited by the lattice resolution. Because
fracture growth lacks an intrinsic size scale (Bonnet and others, 2001), we expect that
this approach reflects the physics of mode I fractures.

The parameter values listed in table 2 and the final extensional strain are chosen
in the calibration process to produce a regular joint pattern in the absence of
cementation. Running the calculations to much larger extension produces irregular,
tear-like cascades that lead to non-convergence of the force-balance solution. Further-
more, at sufficiently high strain, the imposition of matrix-layer indestructibility be-
comes physically unrealistic. We therefore do not consider how the system of fractures
behaves at arbitrarily high strains.

Defining Fractures

Although the position of many fractures is clear by visual inspection of the
distribution of broken springs, the spring-lattice model has no inherent definition of a
fracture. Automatic detection and measurement of fractures therefore requires the
formulation of a criterion to distinguish discrete from diffuse spring breakage. The
choice of this criterion is complicated by the tortuosity of fracture walls (fig. 5).
Furthermore, since we want to model vein spacing, the criterion must consider springs
reattached by cementation. Potential fracture-definitions include (i) lone broken or
cemented springs, (ii) groups of broken or cemented springs that connect across the
layer, and (iii) nodes separated by a given distance or more. Option (i) would include
isolated breaks of anomalously weak springs; these are evidently not analogous to
structures that would be recognized in the field as fractures. Tortuous fracture paths
preclude a simple approach of detecting a single, connected fracture pathway across
the layer, and so make option (ii) problematic. Option (iii)—defining fractures as
nodes separated by a given distance or more—is straightforward and easily comput-
able.

We search for fractures along the middle row of nodes, $j = (N_j - 1)/2$. We define a
fracture as a neighboring node-pair separated by the median node-separation plus two
standard deviations. This test consistently returns a number of fractures close to that
derived by visual assessment of the final lattice pattern.

Statistics on fracture aperture and spacing were compiled from three simulation
runs and are given in table 3; aperture and spacing values are reported normalized to
the initial layer height $H$ and thus reported as S:T ratios. We use the term aperture to
mean the displacement between neighboring lattice nodes that bound the fracture, regardless of whether the intervening space is occupied by cement. Thus our measure is geologically analogous to the \textit{kinematic aperture} of Marrett and others (1999).

**Quantifying Fracture Spacing**

The spacings generated by the present model are well-fit by normal distributions, as predicted by Rives and others (1992) for well-developed fracture patterns. Thus we can define a mean spacing with some variation (irregularity of spacing) represented by the standard deviation. We can then follow previous studies (table 1) and quantify the spacing irregularity using the coefficient of variation, $C_v$, or standard deviation divided by mean.

For statistical evaluation, we compile all fracture spacings into a single population from each group of three simulation runs having the same parameters; we do the same for fracture aperture. We omit the left-most and right-most spacings in each run (that is, the spacings between a fracture and the left or right boundary of the model).

**Without Cementation**

Using $f = 0$ gives a cementation rate of zero (eq. 19). Figure 6 shows the results of a simulation run under this condition, as well as results using $f > 0$. The final pattern for $f = 0$ shows regularly spaced fractures with spacing that is similar to layer height.

Figure 7 illustrates the development of the fracture pattern through time. It shows the change in distance between neighboring node-pairs, along the middle lattice row, between each model time-step. Fracture opening is represented by an abrupt increase in the distance between the nodes on either side of the fracture; nodes near the fracture become closer together in response to the local reduction in tensile stress. Within a narrow time-window that is early in the simulation, five fractures propagate and relieve the tensile stress in the neighboring parts of the lattice. These fractures are evenly distributed along the layer, thus relieving the stress throughout. Later in the simulation, after further extension has again raised the tensile stress, more fractures

<table>
<thead>
<tr>
<th>$K_C$</th>
<th>$f$</th>
<th>$N$</th>
<th>Aperture</th>
<th>Spacing</th>
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</table>

$N$ is the total, combined number of fractures in each of three simulation runs for each set of parameters. Apertures and spacings normalized to layer height $H$. 

<table>
<thead>
<tr>
<th>$K_C$</th>
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<th>Aperture</th>
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appear; these infilling fractures preferentially develop at positions roughly equidistant between the previous generation of fractures, where tensile stress is highest.

Because figure 7 is constructed from the middle row of nodes, it does not illustrate the deformation near the top and bottom of the fracturing layer. Figures 5 and 6 show...
Fig. 7. Development of fractures through time. Node spacing $\delta_{ij} = |x_{i+1} - x_i|$ is the change in distance between each neighboring node-pair along the middle row of the lattice. $\delta_{ij}$ has units of $\Delta_y$. Each model comprises 100 time-steps of equal extension, resulting in a final extension $\varepsilon_{xx} = 0.144$. 
an increase in the density of broken springs near the layer boundaries, including secondary fractures that are rooted at either boundary but do not cross the fracturing layer. This pattern appears to arise because of the unbreakable matrix layers that bound the fracturing layer. The matrix layers diffuse the localized extension associated with the primary fractures. The extension of the matrix layers is also made laterally uniform by the boundary condition at the top and bottom of the domain. This uniform extension transfers tensile stress across the matrix layers into the fracturing layer, resulting in the secondary fractures. The matrix spring constant $K_m$ used in our simulations was the same as the reference value for fracturing-layer stiffness ($K_r$, table 2); using a less-stiff matrix might decrease the tendency to form secondary fractures. Secondary fractures remain trapped near the layer boundaries until sufficient tension has developed across the fracturing layer. Modeling work by Bai and Pollard (2000b) produced similar secondary fractures because flaws near the top and bottom of fracturing layers are more likely to propagate than those near the middle. In that study it was argued that there remains a minimum fracture spacing achievable in layerbound sets, at which the pattern is saturated and no further fractures can infill. Our simulations did not extend the lattice enough to test this claim; we address potential consequences of and for fracture saturation in the Discussion.

Figure 8 shows the $x$-component of normal stress throughout the spring lattice at five different values of layer extension. Fracture opening causes tension relief in the fracturing layer near fracture walls and concentration of tension in matrix layers at fracture tips.

**With Cementation, $K_c = K_r$**

Figure 6 shows that faster cementation produces a closer final fracture spacing. This tendency is quantified in figure 9, which shows the variation in fracture aperture...
and spacing with cementation-rate factor $f$ for the final fracture pattern. There is a transition in both average aperture and spacing over a narrow range of $f$, near 0.6 to 0.8. At slower cementation rates, cement makes little difference to the final pattern, compared to the case of $f = 0$. We call these outcomes *joint-like* behavior. At $f$ greater
than approximately 1, the final apertures and spacings are again mostly insensitive to the cementation rate, showing only gradual decline with faster cementation. This fast-cementation limit represents vein-like behavior. The transition between joint-like and vein-like behavior corresponds to the cementation rate approaching and exceeding the typical opening rate of a fracture.

$C_v$ remains below 1 at all simulated values of $f$, suggesting that spacing is more regular than would be expected for a random arrangement. There is a local maximum in $C_v$ near $f = 0.8$ (fig. 9), although this increase in spatial irregularity is subtle on fracture maps (fig. 5). We tested the increase in $C_v$ for statistical significance using F-tests of spacings, where each spacing in the population is normalized to the population mean. F-tests are sensitive to non-normality, but best-fit normal distribution curves (fig. 9C) show good agreement with modeled spacings. The F-tests indicate that the increase in $C_v$ for $f = 0.8$, relative to $f = 0$ and $f = 5$, is statistically significant to 99 percent confidence.

The simulation output shown in figure 6 (final fracture patterns) and figure 8 (stress maps) are drawn from the first round of simulations. In this round, all simulations are initialized with the same pattern of mechanical heterogeneity of the spring lattice. The divergent fracture patterns indicate that the location of fractures is sensitive to $f$. Figure 5 shows that this divergence emerges early in the discrete phase of deformation. However, for each simulation there is a narrow distribution of fracture aperture and spacing, shown by the error bars in figure 9. This stability reflects the self-organization of the fracture pattern from a disordered initial state to a semi-regular final state under the control of the problem physics and parameters.

Stress maps (fig. 8) show that with cement precipitation, regions of tension relief around fractures are ephemeral, in contrast to the case of no cementation. Subsequent fractures form in the previously relaxed regions, resulting in closer final fracture spacing.

Because crack-seal texture is common in veins and indicates repeated sealing and re-cracking of vein cement during fracture opening, we tracked the number of times each spring broke during layer extension. Figure 10 shows maps of the final spring-lattice for various simulations, with the size and color of each spring corresponding to the number of times it broke during layer extension. With $f = 0$, no re-breaking occurs because springs do not heal. With $K_C = K_r$, springs break as many as three times at fast cementation rates.

With Cementation, $K_C \neq K_r$

The cement that precipitates in veins is commonly of a different composition from the host rock, so the mechanical properties of cement may also be different from those of the host rock. We investigated potential effects of contrasting mechanical properties by increasing and decreasing the spring constant of cement, $K_C$, while holding the spring constant of host rock, $K_r$, constant. We consider one case of weaker cement and two cases of stronger cement in rounds of simulations that use $K_C = \{0.5, 2, 5\}$. The most obvious interpretations of such variations in $K_C$ are contrasts in stiffness between the fracture cement and the host rock. Such contrasts might be expected, for example, if fractures in shales are sealed with calcite. However, we suggest other possible natural analogies in the Discussion.

For these tests we used a moderate ($f = 1$) and fast ($f = 2$) cementation rate. At $K_C < K_r$ and $f = 1$, the final fracture spacing is greater than that for $K_C = K_r$ (fig. 6, 8). This increase in spacing diminishes with increasing $f$. Figure 10 shows that re-cracking of individual broken springs is rare when $K_C = K_r$, although re-cracking increases slightly with increasing $f$. In contrast, decreasing $K_C$ results in substantially more re-cracking events upon individual springs.
Fig. 10. Final spring lattices color-coded according to the number of times each spring has broken.
Increasing $K_c$ relative to $K_r$ makes no statistically significant difference to the fracture pattern (table 3).

**Discussion**

The results presented above show compelling similarity to some features of stretched and fractured sedimentary layers. Furthermore, our model makes predictions about the role of synkinematic cementation in modifying fracture spacing. In this section we discuss these results further. Before that discussion it is important to note that the spring-lattice model and its parameters do not have a unique geological interpretation. The key assumptions that underlie our use of the model are that (i) the physics of the spring lattice with breakable springs is qualitatively similar to the physics of a lithified sedimentary bed, and (ii) that by producing a quantitatively realistic fracture spacing in the absence of cementation, we have calibrated the model to produce meaningful results with the addition of cementation. Further elaboration of the details of these assumptions quickly becomes problematic. For example, there is a potential analogy between lattice nodes and sedimentary grains. Unfortunately, our highest-resolution simulations are still coarser than the grain arrangement of a 30 cm-thick, coarse sandstone bed. In some rocks the mechanical heterogeneity might be dominantly supplied by microcracks. Ideally, such microcracks would be best modeled by seeding the lattice with broken springs. The drawbacks to this approach include the lattice-diagonal propagation and the tendency for such microcracks to fill with cement before bed-scale fractures form.

These caveats aside, the various strengths of spring-lattice models make them a useful tool for exploring hypotheses about rock fracture. As with previous work, cascading spring breakage is here taken to represent fracture in response to applied extension. It is known from fracture mechanics that an incipient crack that reaches critical crack-tip stress intensity can propagate unstably, as the stress required for crack propagation decreases with fracture length (Lawn and Wilshaw, 1975). Accordingly, fractures in the present model tend to propagate instantaneously (unstably) across the fracturing layer, locally relieving tension as a result. In the absence of cementation, relieved tension suppresses nearby fracture initiation, and a regular fracture spacing emerges. Thus our model reproduces the leading-order behavior of a layerbound fracture system as it is currently understood to develop (Schöpfer and others, 2011).

It is, however, the mechanical role of synkinematic cement precipitation that is of interest here. To model cementation, we represent cement that accumulates to span a fracture as a re-growth and re-attachment of broken springs, and we assign a new neutral spring-length such that reattached springs are unstressed when they reconnect. This approach is supported by two common observations of natural cemented fractures. First, fracture cements grow into fluids (Fisher and Brantley, 1992; Bons, 2001; Schulz and others, 2002; Fischer and others, 2009; Becker and others, 2010; Mazzarini and others, 2010; Sample, 2010; Fall and others, 2012), so cement crystals should be free of shear tractions until they span the width of the fracture, however narrow the opening increment. Second, cemented grains re-crack repeatedly during opening (Ramsay, 1980; Laubach and others, 2004; Bons and others, 2012; Hooker and others, 2014), indicating that tension repeatedly develops within the cement. A simple explanation of the genesis of that tension is that cement adheres to both sides of the fracture as the fracture widens in response to externally imposed layer-extension.

**Cementation and Fracture Patterns**

The effect of cementation of fractures on fracture opening and pattern formation is to restore cohesion across fractures; continuing layer-extension will then result in tension in and around cemented fractures. As extension proceeds, faster cementation causes tension to develop more quickly throughout the host rock, so that the regions
between fractures reach the stress required for fracture initiation sooner. Consequently, at a given strain there will be more fractures relative to the case of zero cementation. These cemented fractures will be more closely spaced and narrower because they accommodate a smaller portion of the total strain.

With increasing cementation rate, mean fracture aperture decreases. With zero cement precipitation, the average final aperture is $0.20H$ (recall that $H$ is the height of the fracturing layer). At $f < \sim 0.5$, there is a slow reduction of aperture with increasing cementation to $0.15H$. Beyond a transitional range of roughly $0.5 < f < 1$, apertures plateau at a minimum average aperture of $\sim 0.08H$. This number can be understood in terms of a scenario in which a fracture opens instantaneously and then is immediately cemented; the aperture that becomes stabilized during cementation is that achieved during fracture propagation. This aperture can be obtained by considering the instantaneous aperture of uncemented fractures ($f = 0$). This is approximately the average aperture immediately after the short time interval during which the regularly spaced fracture array forms (for example, at $\varepsilon_{xx}/H = 0.051$, at which the aperture is $\sim 0.06H$—fig. 7). The faster the cementation rate, the sooner fracture widening is resisted by cementation and the closer the final cemented aperture to the instantaneous aperture. The final aperture for arbitrarily high $f$ remains higher than the instantaneous uncemented fracture aperture because the cement stretches with layer extension.

The degree of spacing irregularity ($C_v$, fig. 9) reaches a local maximum near the transition from joint-like to vein-like behavior, $f \sim 0.8$. We find equally high $C_v$ at fast cementation rates ($f = 10$). It may be that spacing irregularity would continue to rise with further increasing cementation rate. However, through all simulations, the pattern remains more regularly spaced than would be expected for random fracture locations. That is, $C_v$ remains less than 1. Indeed, the location of a new fracture among an extant array is not random. The non-fracturing matrix layers, which are bonded to the fracturing layer, tend to homogenize extension within the fracturing layer. The restoration of tension that comes about from the combination of extension and cementation appears to interfere in detail with the locus of nucleation of new fractures, as shown by the statistically significant increase in spacing $C_v$ of cemented versus uncemented fractures.

Previous work quantifying fracture spacing and spacing irregularity (fig. 2) shows that, relative to joint spacing, vein spacing is closer and usually more irregular. Our model qualitatively reproduces these pattern changes and thus supports the idea that the mechanical effects of cement can modify fracture patterns. Quantitatively, the pattern changes produced by cementation in the model are more subtle than those seen in nature. We observe from previous studies that joint S:T typically ranges from 0.5 to 2, whereas vein S:T is commonly as low as 0.1 (fig. 2). In the model we see a transition in S:T from $\sim 1.4$ to $\sim 0.6$ between joint-like versus vein-like behaviors, respectively. Natural joint spacings show $C_v$ between 0.5 and 0.6; natural vein-spacing $C_v$ can exceed 1 (fig. 2). Our model shows increased $C_v$ for all cemented cases relative to the zero-cement case, but does not reproduce $C_v > \sim 0.6$.

Thus in our model, adding cementation during layer extension and fracturing reproduces the qualitative differences between vein patterns versus joint patterns in nature, in terms of fracture-spacing and irregularity. However, the model does not change the original joint patterns to be as closely spaced or clustered as many natural vein sets. It may be that further calibration of higher-resolution lattices would produce greater effects of cementation on fracture patterns. Still, it is important to recall that this model is intended to show the effects of cementation using a purely elastic approach, in isolation; natural vein-formation probably involves other processes that we have not modeled here. For example, there is evidence that natural fractures
propagate subcritically (Savalli and Engelder, 2005), such that an individual fracture
does not cross the bed quickly (unstably) as is the case in the present model.
Therefore, multiple closely spaced fractures might propagate, open, and fill simultane-
ously, before a single fracture traverses the bed and prohibits nearby fractures to form
(Olson, 1993).

Moreover, it has been shown that in some cases, natural fractures open over tens
of millions of years (Becker and others, 2010), so creep processes in the host rock may
also dissipate the stresses around fractures (Brantut and others, 2013), homogenizing
the stress field. Depending on the source of the cement, which is not considered here,
the present model can be thought of as incorporating the precipitation part of
solution-precipitation creep. In a case of thorough dissipation of elastic stress due to
creep, new fractures might indeed form in random locations, resulting in an exponen-
tial distribution of fracture spacings and a $C_v$ near 1 (Gillespie and others, 1999). That
late joints crosscut early veins with little change in orientation where they intersect (fig.
1A) is evidence that the elastic stresses around veins are dissipated over time.
Nonetheless, in addition to inelastic processes, the increased tension resulting from
combined cementation and extension, illustrated by our model, should be taken into
account when trying to fully understand and model the physics behind fracture
opening amid cementation. It is important to note that the effects of cementation on
vein patterns could apply to vein patterns in general, not just to layerbound veins
whose contrasts to layerbound joints are readily apparent (fig. 2).

Re-Cracking of Pre-Existing Fractures

Many natural veins show evidence of repeated re-cracking in the form of crack-seal
texture. In contrast, our simulations in which cement has the same mechanical
properties as the host rock produce little re-cracking. Only when veins are weaker than
the host rock (that is, $K_C < K_r$) does significant re-cracking occur (fig. 10). Irrespective
of mechanical properties, once a fracture forms and is sealed by cement, tension will
re-build across the fracture, at some rate, with further stretching of the layer. However,
the region near the fracture will remain relaxed of tension, relative to the regions that
did not fracture. Consequently, even with cementation, we anticipate that the regions
of the lattice furthest from pre-existing fractures (that is, those regions equidistant
from pre-existing, regularly spaced fractures) will be under the highest tension and
thus most likely to host subsequent fractures.

The aversion to re-cracking can be overcome if the spring constant for cemented
springs, $K_C$, is sufficiently small. In such cases, layer extension preferentially widens
the low-$K_C$ cement, causing those cement springs to reach the critical failure length before
the host-rock springs fail. This is why low-$K_C$ cement tends to re-fracture more
frequently than cement that is mechanically identical to the host rock. Vass and others
(2014) found equivalent results varying the Young’s modulus of cement, and showed
that spring breaking strength had an even greater effect on spring re-fracturability.
Similarly, discrete element modeling by Virgo and others (2013) suggested that
whether a new crack forms along or nearby an extant crack depends on the relative
strengths of the vein material and host rock, as well as the angle between extant veins
and new fractures. Thus, the results of spring-lattice models are consistent with sealed,
layerbound, re-opened fractures remaining sites of mechanical weakness, despite their
cement.

Our modification of $K_C$ most straightforwardly represents cases in which the
cement material is markedly different from that of the host rock. However, the
interface between the host-rock and cement could also supply the weakness necessary
for re-cracking (Caputo and Hancock, 1999). The absence of crystal bonds between
calcite and mudrock was invoked to explain preferential breaking of core samples
along calcite-filled fractures by Gale and Holder (2010). As well, incomplete sealing of
fractures could result in persistent mechanical weaknesses. In principle our model can account for incomplete sealing; locally narrow fracture tips and thin, secondary fractures near the matrix layers are commonly sealed before the wider central parts of veins (fig. 5). Similarly, fast sealing of narrow vein-tips was invoked to explain branching vein tips by Bons and others (2012).

In nature, other processes, besides tapering fracture tips, contribute to partial fracture sealing. Two such processes, not included in the present model, are described by Lander and Laubach (2015). First, there are grain-scale variations in cementation rate, which arise from grain mineralogy, size, and crystallographic orientation with respect to fracture walls. Second, there is a considerable drop in cementation rate once cement overgrowths develop euhedral surfaces (Lander and others, 2008). Cement within wider fractures is more likely to become euhedral because there is a greater fluid-filled distance for such deposits to traverse before the fracture is sealed. Thus larger fractures are less likely to be sealed and more likely to remain planes of weakness. Hooker and others (2012) conjectured that power-law aperture distributions arose because less-complete cementation of wider fractures resulted in natural preferential re-cracking of wider fractures, creating a positive feedback loop between fracture size and growth potential.

**Implications for Fracture Saturation**

In the model of Hobbs (1967), sufficient extension will create infilling fractures of arbitrarily small spacing, owing to bonding between matrix layers and fracturing layers. Thus in that model the fracture pattern never truly saturates; rather, ever more extension is required to create a new set of joints in between extant ones. Alternative model constructions produce true saturation, either from proposed frictional sliding between layers creating a maximum tension transferable by the matrix layers (Price, 1966) or from Poissonian compression related to vertical contraction around fractures (Bai and others, 2000). The present work does not test hypotheses about fracture saturation because of the difficulty in numerical convergence associated with increasing extension, as discussed in the Results section.

However, regardless of whether the pattern would eventually saturate, the effects of cementation would be similar to what we describe because in either case, extension produces widening of fractures. If those fractures are sealed with cement, then tension will develop across them. The effects of cement might be more important in cases of true saturation, because with increasing strain, those systems produce greater widening of existing fractures in the absence of new fractures in between. Furthermore, if saturation is achieved by frictional sliding along bedding planes, then cementation might heal shear fractures along bedding planes, facilitating tension-restoration and further fracturing. But even in the no-saturation case described by Hobbs (1967) we would still expect a closer fracture spacing with increased cementation rate, because cementation modifies the fracture pattern early, before infilling fractures have formed in the zero-cementation case (compare $f = 2$, $f = 0$, fig. 7).

**Conclusions**

We presented a model of fracture cementation and layer extension, in which fractures grow across a single layer. We assumed that cementation proceeds at a constant rate, without consideration of the source of the cement. For cement having identical mechanical properties to the host-rock, faster cementation rates produce thinner and more closely- and irregularly-spaced veins. Such veins typically manifest fewer than three individual opening increments. Decreasing the stiffness of the cement relative to that of the host rock produces veins with multiple re-cracking episodes, and results in wider vein spacing, approaching that of the zero-cement case. These results suggest that during vein growth, cement that spans veins resists fracture
widening and, in combination with further layer extension, restores tension, which helps to account for close vein spacing. Natural re-cracking of extant veins, observed within crack-seal vein-cement texture, implies that veins remain sites of mechanical weakness despite their cement. This model of fully elastic, critically-stressed fracture propagation with cementation does not account for the observation that many natural veins show random or systematically clustered spatial arrangements. Therefore creep processes likely also affect natural vein patterns.

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