Plate-driven mantle dynamics and global patterns of mid-ocean ridge bathymetry

Samuel M. Weatherley and Richard F. Katz
Department of Earth Sciences, University of Oxford, South Parks Road, Oxford OX1 3AN, UK
(samw@earth.ox.ac.uk)

Global observations of mid-ocean ridge (MOR) bathymetry reveal a correlation between the difference in axial depth across ridge offsets and ridge migration. If upwelling and melt production rates are asymmetric across a ridge axis, three-dimensional (3-D) melt focusing and different crustal thickness across offsets may account for the observed differences in axial depth. In this article we use a 3-D numerical model to constrain the flow and thermal structure of a ridge-transform-ridge plate boundary. By coupling a model of melt focusing to our simulations we generate predictions of crustal thickness and axial depth change across offsets of different lengths. These predictions are consistent with the morphological changes observed along the global MOR system. In making these predictions we produce new constraints on the scale of melt focusing at mid-ocean ridges and on the extent of melt redistribution at the ridge axis. Results from our simulations also suggest that plate-induced mantle dynamics and melt focusing beneath a migrating MOR may produce global, systematic variations in the geochemistry of axial lavas.

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1. Introduction

Earth’s mid-ocean ridges (MORs) are a striking geological feature resulting from plate-mantle interaction. As oceanic plates spread apart, hot asthenospheric mantle upwells and melts adiabatically, supplying the spreading center with material to generate new crust. On a global scale, the physical and petrologic properties of oceanic crust generated at spreading centers are approximately independent of plate kinematics, for fast to slow spreading ridges. These similarities suggest that the fundamental processes governing plate-mantle interaction are uniform throughout the globe. On a regional scale, however, more subtle differences in the physical and chemical properties of oceanic crust become apparent. In this study we use these regional differences to investigate the details of plate-mantle interaction.

Global observations of MOR bathymetry by Carbotte et al. [2004] reveal a correlation between the difference in axial depth across ridge offsets and the direction of ridge migration relative to the fixed hot spot reference frame. At the majority of offsets, ridge segments leading with respect to the direction of ridge migration are shallower than trailing segments. The systematic connection between axial depth change across offsets and plate
kinematics suggests that an explanation might be found in plate motion–induced mantle dynamics. Differences in axial depth across offsets are thought to arise from differences in the volumes of melt being delivered to the leading and trailing ridge segments [Carbotte et al., 2004; Katz et al., 2004]. However, the three-dimensional nature of melt generation and focusing in the vicinity of MOR offsets requires further investigation.

[6] Katz et al. [2004] used a 2-D numerical model to simulate asymmetric mantle flow beneath a migrating ridge. These simulations quantified the conceptual model proposed by Carbotte et al. [2004]. Figure 2 shows example output from a 2-D simulation after Katz et al. [2004]. The mantle flow pattern beneath a migrating MOR is shown in Figure 2a; Figure 2b shows the component of flow induced by ridge migration. To generate predictions of axial depth changes across offsets of different lengths, the authors coupled a simple parametric model of melt focusing to their simulations.

[7] The parametric melt focusing model used by these authors assumes that melt focusing regions are rectangular-shaped in map view and are located at the end of ridge segments. For their experiments, the dimensions of the focusing regions were fixed for all offsets and spreading rates. Melt from these focusing regions was distributed evenly over the first 1 km of the ridge segments, in keeping with the distance over which Carbotte et al. [2004] make their observations. Predictions of axial depth asymmetry generated by the 2-D model are consistent with the observations made by Carbotte et al. [2004].

[8] Although Katz et al. [2004] generate reasonable predictions of axial depth differences across offsets, their model does not consider the fundamentally 3-D nature of transform faults. The mantle flow pattern and thermal structure is two-dimensional and constant along the entire length of each ridge segment. In nature, ridge offsets such as transform faults juxtapose cold, thick lithosphere against warm ridge segments. This temperature difference
modifies the thermal structure of the lithosphere and mantle surrounding the offset. Because the thermal structure of the lithosphere and upper mantle close to offsets is three dimensional, mantle flow and melt focusing cannot be fully investigated with a 2-D model. Close to offsets, changes in the temperature structure of the mantle with offset length and spreading rate will affect the shape and dimensions of the focusing regions. Furthermore, at the ridge axis, processes such as melt flow through the porous mantle and cracks are likely to redistribute melt over a substantial section of the ridge axis. These deficiencies of the 2-D simulations can be overcome by extending the model into three spatial dimensions.

In this paper we use 3-D, steady state numerical simulations to investigate the dynamics of the mantle and melt focusing beneath a migrating MOR. We confirm that the magnitude of asymmetry generated solely by ridge migration is sufficient to explain the observations made by Carbotte et al. [2004]. However, the behavior of our predictions of axial depth asymmetry across offsets is different to that predicted by the 2-D simulations of Katz et al. [2004]. We show that the difference in bathymetry between two adjacent ridge segments depends on the difference in melt volume generated in the focusing regions and the distance over which melt is redistributed at the ridge.

The remainder of this paper is divided into three sections. In section 2 we describe the construction of the numerical model that solves for passively driven, incompressible, creeping mantle flow. Section 2 also details the parameterization of mantle melting, melt focusing, and melt redistribution that we couple to the numerical model. Section 3 shows how predictions of axial depth differences generated by our simulations compare against global MOR data for a range of offsets and spreading rates. In section 4 we discuss how the spreading rate, rate of ridge migration, and geometry of a MOR system influence asthenospheric flow, and to what extent they control differences in axial depth across an offset. Further to this we examine the influence of a viscoplastic rheology on mantle flow and melting beneath a migrating MOR. We also consider how additional geophysical, geochemical, petrological, and numerical studies can better constrain the melt focusing and redistribution processes at MORs.

2. Model Construction

The model is based on a set of coupled partial differential equations to describe incompressible, steady state, passively driven solid mantle flow. We expect that these equations capture the upwelling behavior of mantle beneath migrating MORs. By coupling the computed solid mantle flow field to an existing parameterization of mantle melting [Katz et al., 2003] we determine the melting rate throughout the simulation domain. In the final step of the computation, we apply a model of 3-D melt focusing to the simulations to generate predictions.

Figure 2. Example output from the 2-D simulations by Katz et al. [2004]. (a) The mantle flow pattern beneath a migrating ridge and (b) the component of mantle flow induced by ridge migration are compared. Figure 2a shows a solid mantle flow pattern (white arrows) from 2-D simulation with half spreading rate $U_o = 4$ cm/yr and ridge migration rate $U_r = 4$ cm/yr. The ridge is migrating to the left. Colored background shows viscosity in Pa s. Figure 2b shows the component of mantle flow caused by ridge migration. White boxes show the regions within the leading (L) and trailing (T) minor focusing regions.
The following system of coupled partial differential equations governs passive solid mantle flow.

\[ \nabla \cdot \mathbf{v} = 0, \]

\[ \nabla P = \nabla \cdot [\eta (\nabla \mathbf{v} + (\nabla \mathbf{v})^T)], \]

\[ \mathbf{v} \cdot \mathbf{\nabla} \Theta = \kappa \nabla^2 \Theta. \]

The system includes expressions for the three-dimensional mantle velocity, pressure, dynamic viscosity, and temperature, respectively. The equation of motion (1) expresses the conservation of momentum for an incompressible fluid, while the equation of state (2) maintains the balance of pressure due to the gradient of the velocity field. The temperature equation (3) describes the heat flux due to the strain rate of the mantle.

Table 1. Boundary Conditions\(^a\)

<table>
<thead>
<tr>
<th>Boundary</th>
<th>Variable</th>
<th>Boundary Condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>( z = 0 )</td>
<td>( u )</td>
<td>( u(x, y, 0) = U_0(x, y) ) where ( U_0 ) is the prescribed plate motion</td>
</tr>
<tr>
<td></td>
<td>( v )</td>
<td>( v = 0 )</td>
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<tr>
<td></td>
<td>( w )</td>
<td>( w = 0 )</td>
</tr>
<tr>
<td></td>
<td>( P )</td>
<td>BC does not influence the interior of the domain</td>
</tr>
<tr>
<td></td>
<td>( T )</td>
<td>( T = 0 )</td>
</tr>
<tr>
<td>( z = Z )</td>
<td>( u )</td>
<td>( u = U_0 ) where ( U_0 ) is the ( x ) component of ridge migration</td>
</tr>
<tr>
<td></td>
<td>( v )</td>
<td>( v = 0 )</td>
</tr>
<tr>
<td></td>
<td>( w )</td>
<td>( w = 0 )</td>
</tr>
<tr>
<td></td>
<td>( P )</td>
<td>BC does not influence the interior of the domain</td>
</tr>
<tr>
<td></td>
<td>( T )</td>
<td>Dimensionless ( T ) set equal to dimensionless mantle potential temperature</td>
</tr>
<tr>
<td>( x = 0, X )</td>
<td>( u )</td>
<td>Satisfies discretized form of equation (1)</td>
</tr>
<tr>
<td></td>
<td>( v )</td>
<td>( v = 0 )</td>
</tr>
<tr>
<td></td>
<td>( w )</td>
<td>( w = 0 )</td>
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<td></td>
<td>( P )</td>
<td>BC does not influence the interior of the domain</td>
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<tr>
<td></td>
<td>( T )</td>
<td>( T = 0 )</td>
</tr>
<tr>
<td>( y = 0, Y )</td>
<td>( u )</td>
<td>No shear stress</td>
</tr>
<tr>
<td></td>
<td>( v )</td>
<td>No normal stress</td>
</tr>
<tr>
<td></td>
<td>( w )</td>
<td>No shear stress</td>
</tr>
<tr>
<td></td>
<td>( P )</td>
<td>BC does not influence the interior of the domain</td>
</tr>
<tr>
<td></td>
<td>( T )</td>
<td>( T = 0 )</td>
</tr>
</tbody>
</table>

\(^a\)The domain size is \( x \in [0, X], y \in [0, Y], z \in [0, Z] \). The \( x \) dimension is parallel to the spreading rate vector, \( y \) dimension is parallel to the ridge axes, and the \( z \) dimension is depth.

of the difference in crustal thickness and axial depth across an offset. Each of these steps is described in more detail below.

2.1. Solid Mantle Flow

The following system of coupled partial differential equations governs passive solid mantle flow.

\[ \nabla \cdot \mathbf{v} = 0, \]

\[ \nabla P = \nabla \cdot [\eta (\nabla \mathbf{v} + (\nabla \mathbf{v})^T)], \]

\[ \mathbf{v} \cdot \mathbf{\nabla} \Theta = \kappa \nabla^2 \Theta. \]

2.2. Boundary Conditions

This system of equations is closed with a constitutive relation for mantle viscosity. The mantle flows by diffusion and dislocation creep; the creep mechanism giving rise to the highest strain rate controls the local viscosity of the mantle. We define the viscosity, \( \eta \), as

\[ \eta = \left( \frac{1}{\eta_{\text{dinf}}} + \frac{1}{\eta_{\text{disl}}} \right)^{-1}. \]

where \( \eta_{\text{dinf}} \) is the viscosity due to pressure- and temperature-dependent diffusion creep and \( \eta_{\text{disl}} \) is the viscosity due to pressure-, temperature-, and strain rate–dependent dislocation creep. We combine \( \eta_{\text{dinf}} \) and \( \eta_{\text{disl}} \) in this way to capture transitions in the dominant creep mechanism within the simulation domain. The definitions of \( \eta_{\text{dinf}} \) and \( \eta_{\text{disl}} \) are as follows:

\[ \eta_{\text{dinf}} = A_{\text{dinf}} \exp \left( \frac{E^*_{\text{dinf}} + PV^*_{\text{dinf}}}{RT} \right) \]

\[ \eta_{\text{disl}} = A_{\text{disl}} \exp \left( \frac{E^*_{\text{disl}} + PV^*_{\text{disl}}}{nRT} \right) \epsilon \frac{\mu^*}{\mu^0} \]

where \( \eta_{\text{disl}} \) is the viscosity due to pressure- and temperature-dependent diffusion creep, and \( \eta_{\text{dinf}} \) is the viscosity due to pressure-, temperature-, and strain rate–dependent dislocation creep. \( A \) is a proportionality constant, \( E^* \) is activation energy, \( P^* \) is an activation volume, \( R \) is the gas constant, \( T \) is temperature, and \( n \) is the power law exponent [Karato and Wu, 1993; Hirth and Kohlstedt, 1996]. \( \epsilon \frac{\mu^*}{\mu^0} \) is the second invariant of the strain rate tensor, providing a measure of the intensity of strain rate.

In our model the lithosphere consists of material with sufficiently high viscosity to be considered rigid. For the purpose of this paper, in each vertical column of grid cells, the base of the lithosphere is defined where \( |\nabla \Theta| \) is maximum.
ridge migration in a direction perpendicular to the
ridge axis \( U_r \). They show that the mean ratio of \( U_r \)
to \( U_0 \) is 0.95 with a standard deviation of 0.33. For
simplicity, in this study we take \( U_r \) to equal \( U_0 \).
Because the equations are solved in a reference
frame fixed to the migrating ridge, the velocity
field on the top boundary is set to that of sym-
metrically spreading plates, and \( U_r \) is imposed on
the bottom boundary.

[17] The bottom boundary of the domain is intended
to correspond to the base of the asthenosphere. For
this investigation we take the base of the astheno-
sphere to be rigid. A large drop in horizontal
mantle velocity with increasing depth is expected
across the base of the asthenosphere [Richards et al.,
2003; Panza et al., 2010]. This indicates that
the component of mantle flow induced by ridge
migration will be concentrated below the top of
the lithosphere-asthenosphere boundary and above the
base of the asthenosphere. The radially anisotropic
shear velocity structure of sub-MOR mantle may
indicate that the depth of the base of the astheno-
sphere is 200 km [Nettles and Dziewoński, 2008].
However, large variations in the thickness of the
asthenosphere occur across the globe. For our
simulations we use an asthenospheric depth of
300 km. Katz et al. [2004] show that the difference
in axial depth predicted by the 2-D simulations
scales inversely with the asthenospheric thickness.

2.3. Computational Methods

[18] We iteratively solve a finite volume discretiza-
tion of equations (1)–(3) and the complete set of
boundary conditions using a Newton-Krylov solver
provided by the Portable, Extensible Toolkit for
Scientific Computing (PETSc) [Balay et al., 1997;
Katz et al., 2007; Balay et al., 2009]. The dislo-
cation creep term in equation (4) introduces strong
nonlinearity into the system, and presents an addi-
tional challenge in generating numerical solutions
to the governing equations.

[19] To handle the nonlinearity in equation (4) we
adopt a continuation method, forcing the variation
in viscosity to go from zero to the full predicted
variance over a set of iterations of the nonlinear
solve [Knepley et al., 2006; Katz et al., 2007]. To
smoothly control the variation in viscosity, we set
an upper limit of \( 10^{23} \) Pa s and use this limit to
normalize \( \eta \) to a range between zero and one. The
viscosity field \( \eta^\star \) used on the \( m \)th iteration of the
continuation loop is given by

\[
\eta^\star = \eta^{\alpha_m} \quad \alpha_m \in [0, 1]
\]

where \( m = 1, 2, \ldots, M \). In the first iteration \( \alpha_1 = 0 \).
This yields the solution to the isoviscous case,
which is then used as the starting guess for the next
step, where \( \alpha_2 > \alpha_1 \). The continuation loop ends at
\( \alpha_M = 1 \) which corresponds to the full predicted
variance in viscosity.

2.4. Melting and Melt Migration

[20] We compute the rate of melt production using
a parameterization of mantle melting by Katz et al.
[2003]. This parameterization expresses melt frac-
tion as a function of pressure, temperature, water
content, and modal clinopyroxene. For a given
mantle potential temperature, the parameterization
predicts that the adiabatic productivity \( dF/dz_S \) is
approximately constant. The adiabatic melting rate
\( \Gamma \) kg/m\(^3\)/yr is then calculated as

\[
\Gamma = \rho_m W \left. \frac{dF}{dz} \right|_S
\]

where \( \rho_m \) is the density of the mantle, \( W \) is the ver-
tical velocity from the solution of equations (1)–(3),
and \( S \) is entropy. We use equation (8), the solidus
of Katz et al. [2003], a mantle potential tem-
perature of 1300°C, and an adiabatic productivity of
0.4%/km to calculate the melting rate at each grid
cell in the domain. If ridge migration causes any
perturbation of \( W \), the melting rate will be asym-
metric across the ridge axis.

[21] We assume that the melt from each grid cell
percolates vertically through the mantle to the top
of the melting region. The upper surface of the
melting region is an impermeable boundary that
slopes upward toward the ridge axis. Melt migrates
uphill along this boundary on streamlines that fol-
low the steepest local slope of the surface, that is

\[
\left( i \frac{\partial h}{\partial x} + j \frac{\partial h}{\partial y} \right) \times ds = 0
\]

where \( h \) is the depth to the top of the melting region
and \( ds \) is an element of arc length along the stream-
line. This model of melt migration was first
proposed by Sparks and Parmentier [1991] and has
been used in subsequent studies of 3-D melt
migration [Sparks et al., 1993; Magde et al., 1997;
Magde and Sparks, 1997; Gregg et al., 2009]. To
determine where along the ridge axis melt is
extracted we define a narrow, 2 km wide box, that
surrounds the ridge axis and project it onto the top
of the melting region. At the point where a
streamline crosses into this box, the melt carried
along that trajectory is extracted and accumulated at that position on the ridge.

[22] Although melt focusing is thought to be an efficient process, complete extraction of melt from the mantle is unlikely. Petrological studies of abyssal and mantle peridotites [Warren and Shimizu, 2010; Dick et al., 2010; Seyler et al., 2001, 2007; Niu, 1997], active near-ridge seamounts [Niu and Batiza, 1991; Batiza et al., 1990], numerical simulations of mantle dynamics [Chen, 1996; Ghods and Arkani-Hamed, 2000; Katz, 2008], geophysical experiments [Lizarralde et al., 2004], and integration of geophysical and geochemical observations [Faul, 2001] all indicate that between 1% and 40% of the melt generated within the melting region is retained in the mantle.

[23] In our models, all of the melt extracted at a ridge segment originates from inside a focusing region. We limit the maximum horizontal extent of the focusing regions from the ridge axis, and hence the fraction of melt extracted at the ridge, by specifying a focusing distance. We assume that the focusing distance increases with spreading rate to maintain a constant crustal thickness of 6 km at an on-axis point 70 km away from the transform fault.

[24] Figure 3a shows predicted melt migration trajectories for a simulation with \( U_r = 2 \) cm/yr and an offset of 40 km; colors and contours show the solidus depth. The solidus depth increases close to the transform fault, deflecting melt away from the end of the ridge segment. This leads to on-axis bunching of melt trajectories 10–30 km away from the transform fault.

[25] Along-axis profiles of melt accumulation rate resulting from the melt focusing process described in Figure 3a are shown in Figure 3b. At sufficiently large distances from the transform fault, the crust has a constant thickness of 6 km. Close to the transform fault, deflection of melt trajectories causes large variation in crustal thickness. In nature, such large variations are not seen; for intermediate to fast spreading rates crustal thickness is nearly constant along the length of ridge segments. The constancy of oceanic crustal thickness in nature suggests that along-axis redistribution of melt through cracks and porous mantle works to smooth the initial distribution of melt caused by melt focusing [Korenaga and Kelemen, 1997; Magde et al., 2000; Kelemen et al., 2000; Singh et al., 2006a].

[26] In Figure 3b, solid dots indicate the positions at which the gradient of the crustal thickness profiles first departs significantly from zero. The distance between these points and the offset for each simulation is the redistribution distance \( \zeta \). (c) Variation in \( \zeta \) as a function of transform fault length.

Figure 3. Summary of steps needed to focus melt to the ridge and redistribute it along the ridge axis. (a) Map showing the relationship between solidus depth, focusing region shape and melt focusing trajectories. White streamlines show melt focusing trajectories. Black lines outline the geometry of the ridge system. Magenta lines mark the perimeters of the focusing regions. Colored contours show depth to the top of the melting region. Simulation has \( U_r = 2 \) cm/yr and an offset of 40 km. The ridge is migrating to the left. (b) Profiles showing along-axis variation in crustal thickness for simulations with \( U_r = 1, 2, 4, \) and 6 cm/yr and an offset of 40 km. Solid dots show the positions at which the gradient of the crustal thickness profiles first departs significantly from zero. The distance between these points and the offset for each simulation is the redistribution distance \( \zeta \). (c) Variation in \( \zeta \) as a function of transform fault length.
2.5. Compensation of Mid-Ocean Ridge Crest Topography

[27] To compare the simulation results against data from the global MOR system we must convert the difference in crustal thickness, computed by the melting and melt migration parameterization, into a difference in axial depth. Spectral studies of MOR topography and gravity suggest that MOR topography is supported by flexure of the oceanic lithosphere [Cochran, 1979; McNutt, 1979; Bowin and Milligan, 1985]. In the flexural model of isostasy the oceanic lithosphere is assumed to behave as a perfectly elastic material. The rigidity of the plate is given by the flexural rigidity, $D$,

$$D = \frac{ET_e}{12(1-\nu^2)},$$

where $E$ is Young’s modulus, $T_e$ is the elastic thickness of the plate, and $\nu$ is Poisson’s ratio. In the limit $T_e \to 0$ the lithosphere has no elastic strength and the flexural model reduces to Airy isostasy. In the limit $T_e \to \infty$ the plate becomes rigid and does not flex. For dynamic processes, however, $T_e$ approximately corresponds to the depth of the 600°C isotherm [Watts, 2007]. The flexure ($t - d$) generated by periodic loading of an elastic plate is

$$t - d = d \cos(ky)_0 A \phi_e,$$

where $A$ is the Airy isostatic response,

$$A = \frac{\rho_e - \rho_m}{\rho_m - \rho_c},$$

and $\phi_e$ is the flexural response function,

$$\phi_e = \left[ \frac{DK^4}{(\rho_m - \rho_c)g} + 1 \right]^{-1}.$$

Here, $t$ is the crustal thickness, $d$ is topography, $y$ is the distance along the ridge axis, $k = \pi / \zeta$ is the wave number of the load in the along-ridge direction, and $\rho_m$, $\rho_c$, $\rho_w$ are the densities of the mantle, oceanic crust, and seawater, respectively. The flexural response function $\phi_e$ modifies the Airy isostatic response $A$ so as to represent flexure.

[28] From equation (11), the difference in axial depth, $\Delta d$, between two adjacent ridge segments supported by flexure is related to the difference in crustal thickness, $\Delta t$, by

$$\Delta d = C \Delta t,$$

where

$$\Delta t = \int_0^{-y_0} \gamma t \, dy - \int_0^{y_0 + \zeta} \gamma t \, dy,$$

the isostatic compensation function $C$ is

$$C = [1 + A \phi_e]^{-1}.$$
is different, then there is a difference in crustal production and $|\Delta d| > 0$. Because the minor focusing regions do not terminate at the transform fault and instead extend some way beyond the ridge axis, they are different to those proposed by Carbotte et al. [2004] and Katz et al. [2004].

To calculate the behavior of $\Delta d$ as a function of transform fault length and $U_0$ when $U_0 = U_r$, we use suites of simulations that maintain constant $U_0$ and vary offset length. Figure 5 shows predictions of the difference in axial depth $\Delta d$ generated from four suites of simulations using equation (8). Also shown in Figure 5 are data from the global MOR system [Carbotte et al., 2004] for transform offsets only. These measurements are made by comparing the axial depth of leading and trailing ridge segments. The data are averaged over a 1 km window.

Predictions and observations of $\Delta d$ are for a range of half-spreading rates between 1 and 7 cm/yr and offsets between 0 and 170 km. The focusing distance is 35 km for the suite of simulations with $U_r = 1$ cm/yr, 48 km for the suite with $U_r = 2$ cm/yr,
64 km when \( U_r = 4 \) cm/yr, and 73 km when \( U_r = 6 \) cm/yr. In each case the focusing distance is less than the width of the melting region, hence peripheral melts are not extracted. We find that the simulations using an asthenospheric thickness of 300 km best fit global ridge data. The amplitude of the difference in crustal thickness predicted by our simulations results scales approximately inversely with the asthenospheric thickness.

For fast spreading rates \( (U_0 > 4 \) cm/yr), the model predicts that peak asymmetry in axial depth occurs at offsets of less than 20 km. This distance increases as \( U_0 \) decreases. For slow spreading rates \( (U_0 < 1 \) cm/yr), \( \Delta d \) reaches a peak at offset lengths greater than 60 km. In general, the MOR data show two broad trends. First, the amplitude of the data increases with spreading rate. Second, the amplitude of the data decreases with increasing offset.

The simulations results are consistent with these trends.

4. Discussion

4.1. Three-Dimensional Melt Generation, Focusing, and Redistribution

The simulation results can be understood by considering the component of mantle flow induced by ridge migration. From here onward we refer to this component as the perturbation flow. The vertical component of the perturbation flow \( W' \) is of particular importance. As implied by equation (8), the behavior of \( W' \) in the minor focusing regions controls the magnitude of the bathymetric asymmetry, \( \Delta d \).

At sufficiently large distances from the transform fault, the geometry of the lithosphere-asthenosphere boundary (LAB) and behavior of mantle flow is two-dimensional. Here, the LAB curves upward beneath the ridge segments and the perturbation flow has a vertical component of velocity with upwelling on the leading side of the ridge and downwelling on the trailing side. With increasing depth, the loci of maximum \( |W'| \) for each horizontal row of grid cells moves away from the ridge along the bold lines shown in Figure 6a. Katz et al. [2004] show that the slope of these extremal lines of \( |W'| \) varies with spreading rate. This indicates that far from the transform fault, where mantle flow is two-dimensional, the shape of the LAB controls the upwelling behavior of the perturbation flow.

Close to the transform fault the shape of the LAB is three dimensional. Mantle flow in this vicinity has three components of velocity and the control exerted by the LAB on the perturbation flow is not clear. To determine the influence of the LAB on the perturbation flow, we extend the concept of the extremal lines shown in Figure 6a into three dimensions and consider the relationship between the depth to the LAB and the depth to extremal surfaces of \( |W'| \). If the LAB controls the perturbation flow, the depth to these extremal surfaces should show a clear relationship to the depth to the LAB.

Figure 6b shows that the depth to the extremal surfaces is correlated with the depth to the LAB. This indicates that the three-dimensional shape of the LAB controls the asymmetric, passive, plate-driven upwelling and melting beneath a migrating MOR system.
The offset length and spreading rate principally control the shape of the LAB and melting region. Consequently, these two parameters also influence the shape of the focusing regions. Figure 7 shows how the shape of the focusing regions and melting behavior varies for a suite of simulations with $U_r = 1 \text{ cm/yr}$ and different values of offset. Also shown are the traces of the extremal surfaces where they intersect the bottom of the melting region. The relative spatial positions of the extremal surfaces and minor focusing subregions lends itself to a more detailed explanation of the simulation results.

In the 2-D study by Katz et al. [2004], melt is distributed over a constant along-axis distance at the ridge. Consequently, $\Delta d$ is a maximum when the difference in excess melt production in the focusing regions $\Delta V$ is also a maximum. The magnitude of $\Delta d$ depends on the ratio of $\Delta V$ to the volume of space created at the ridge by seafloor spreading. Katz et al. [2004] found the offset of maximum excess melt production to be approximately constant at 50 km. For 2-D simulations, the offset of maximum excess melt production corresponds to the offset at which the extremal lines intersect the base of the melting region. In other words, when the excess melt production is a maximum, the minor focusing regions are centered above the locus of largest $|W'|$ within the melting region.

Results from our 3-D simulations (Figure 5) predict that the offset of peak $\Delta t$ is dependent on spreading rate. Figure 5 shows that the offset of peak $\Delta t$ decreases with increasing spreading rate. In contrast to Katz et al. [2004], the distance over which melt is distributed at the ridge axis, $\zeta$, varies with spreading rate and offset length (Figure 3). The offset of peak $\Delta t$, therefore, does not necessarily coincide with the offset of maximum excess melt production but depends on the ratio $\Delta V/\zeta$. When $\Delta V/\zeta$ is greatest, the difference in crustal thickness between two adjacent ridge segments is a maximum. The isostatic compensation function $C$ given in equation (16) converts $\Delta t$ into a difference in axial depth, $\Delta d$. When $C\zeta$ is greatest, $\Delta d$ is a maximum. Figure 5 shows that, for a given spreading rate, there is little difference between the offset of peak $\Delta t$ and peak $\Delta d$.

The results in Figure 5 show significant asymmetry about their peak. The component of melting induced by ridge migration is a maximum on the extremal surfaces. It changes most rapidly with horizontal distance on the ridge side of the surfaces. Therefore, $\Delta V$, and thus $\Delta t$ and $\Delta d$, changes most rapidly with offset for offsets less than that of peak $\Delta d$.

Figure 5 shows that $\Delta d$ increases with spreading rate. In plate-driven flow, faster spreading rates drive faster upwelling. In our simulations $U_r$ is equal to $U_0$, and thus the difference in melt production rate between the two minor focusing regions increases with $U_0$, causing $\Delta d$ to increase with $U_0$.

Although the simulation results shown in Figure 5 describe the general trends of the global MOR data, the data are distributed widely about the simulation results. With such a small number of data points it is difficult to assess how well our
simulations and choice of parameters explain axial depth differences along the global mid-ocean ridge system. Furthermore, local, idiosyncratic, geological processes may induce large scatter in the data.

4.2. Constraining Melt Redistribution Processes

Along mid-ocean ridge systems, melt must be focused from a wide, partially molten region to a narrow zone immediately beneath a ridge segment [Kelemen et al., 2000]. To generate new crust, this melt must be tapped and redistributed on the scale of a ridge segment.

Geophysical observations of the MOR system offer an insight to the physical processes involved in melt redistribution. Seismic studies of fast spreading mid-ocean ridge systems suggest that melt is redistributed in the narrow zone beneath ridge segments to form axial magma chambers [Collier and Singh, 1998; Kent et al., 2000; Nedimovic et al., 2005; Singh et al., 2006a, 2006b]. These studies indicate that axial magma chambers for some fast and intermediate spreading rate ridges have a thickness of a few hundred meters and extend nearly continuously along the length of the ridge segment. Seismic reflections [Singh et al., 2006a] from the axial magma chamber beneath the 09°N overlapping spreading center of the East Pacific Rise suggest that melt supply is enhanced beneath the ends of ridge segments. To form such magma chambers, melt must be supplied locally, rather than by large-scale crustal redistribution [Macdonald et al., 1988; Wang et al., 1996]. Field studies suggest that local melt redistribution processes include melt flow through cracks [Singh et al., 2006b], pipe-like features [Magde et al., 2000], and the porous mantle [Kelemen et al., 2000].

Seismic and gravity studies of slow spreading centers show that crustal thickness varies along the length of MOR segments. The amplitude of inferred along-axis variation in crustal thickness decreases with increasing spreading rate [Lin and Phipps Morgan, 1992]. For slow spreading ridges, the crust is usually thickest at the center of the segment and thins toward the ends [Kuo and Forsyth, 1998; Lin et al., 1990; Lin and Phipps Morgan, 1992; Tolstoy et al., 1993; Detrick et al., 1995; Escartín and Lin, 1995; Hooft et al., 2000; Canales et al., 2003; Planert et al., 2009]. This feature has been attributed to enhanced melt delivery at segment centers. Figure 3a shows that the shape of the melting region deflects melt away from the end of the ridge segments. The simulations predict that the variation in crustal thickness prior to melt redistribution decreases with increasing spreading rate. This behavior is common to other numerical studies of melt focusing that use the melt migration model of Sparks and Parmentier [1991] [Magde et al., 1997; Magde and Sparks, 1997; Gregg et al., 2009]. For ridge segments that are tens of kilometers long, this behavior may lead to enhanced melt supply and thicker crust at segment centers. Bell and Buck [1992] note that slow spreading ridges with thick crust do not show large crustal thickness variation on a segment scale, suggesting that melt redistribution may be a function of crustal thermal structure.

Although geophysical investigations can constrain melt redistribution processes on a large scale, the scale and extent of these processes may be studied more intimately using geochemical and petrological techniques. Geochemical variations in ridge axis lavas can yield information about the source region and geological history of the parent magma. Petrological mapping of oceanic basalts at MORs indicates that igneous rocks formed close to ridge offsets have a deeper source region and lower extent of melting than those elsewhere along the ridge segment [Langmuir and Bender, 1984; Langmuir et al., 1986; Reynolds and Langmuir, 1997]. In samples taken along the axis of the north East Pacific Rise, the most enriched geochemical signals are found consistently at the ends of leading ridge segments [Carbotte et al., 2004]. These excursions peak very close to the offset and typically decay rapidly with distance along the ridge axis.

Our simulations suggest that geochemical variations can exist between the melt focused from the leading and trailing focusing regions. Unless the direction of ridge migration is parallel to the ridge trace, the leading focusing region samples relatively deep mantle in advance of the trailing focusing region. Consequently, melts produced in the leading minor melt focusing region will be among the most enriched and compositionally diverse. In contrast, because the trailing minor focusing region lies in the wake of the leading ridge segment, melt generated in this trailing subregion will have smaller variations in trace element geochemistry. The amplitude of these variations will be greatest when the direction of ridge migration is perpendicular to the ridge axis. The amplitude will decrease with the angle between the direction of ridge migration and the ridge axis. Hence our model is compatible with the geo-
chemical observations made by Carbotte et al. [2004].

4.3. Mantle Rheology
[38] Predictions of mantle dynamics and thermal structure generated by our simulations depend largely on the assumed mantle rheology. For this study we assume that the mantle is a non-Newtonian fluid that deforms by combined diffusion and dislocation creep. Katz et al. [2004] show that this choice of rheology improves the fit between simulation results and global MOR data over diffusion creep alone. Other numerical experiments of mantle flow near transform faults by Behn et al. [2007] and Gregg et al. [2009] explore the importance of using a viscoplastic rheology. They show that simulations with a viscoplastic rheology, over those with a constant or temperature-dependent viscosity, better fit geophysical and geochemical observables.

[39] Figure 3 demonstrates the importance of the mantle thermal structure to melt focusing. The experiments by Behn et al. [2007] and Gregg et al. [2009] predict that a viscoplastic rheology tends to localize deformation around the ridge segments and transform faults. This causes the mantle around the transform fault to be warmer and reduces the along-axis distance over which melt is focused away from the offset.

[50] To examine the influence of a viscoplastic rheology on our simulations we define a viscoplastic rheology:

\[
\eta = \left( \frac{1}{\eta_{\text{diff}}} + \frac{1}{\eta_{\text{disl}}} + \frac{1}{\eta_{\text{bsa}}} \right)^{-1},
\]

where \( \eta_{\text{bsa}} \) is a brittle strength approximation using Byerlee’s law. The viscosity associated with brittle failure \( \eta_{\text{bsa}} \) is given by [Chen and Morgan, 1990]

\[
\eta_{\text{bsa}} = \frac{\tau}{2\sqrt{2}\mu g},
\]

in which brittle strength is approximated by a friction law:

\[
\tau = \mu \rho g z + c_0.
\]

Here, \( \mu \) is the frictional coefficient (0.6), \( g \) is the acceleration due to gravity, and \( c_0 \) is the cohesion (10 MPa).

[51] Figure 8 compares two identical suites of simulations with \( U_r = 2 \) cm/yr, except that one (shown in pink) uses the non-Newtonian viscosity defined in equation (4), and the other (shown in blue) assumes the viscoplastic rheology given in
equation (17). For the viscoplastic simulations, the focusing distance is 48 km. This returns a crustal thickness of 6 km at an on-axis point 70 km from the transform fault. Comparison of Figure 8a with Figure 3a demonstrates that the of the morphology of the melting region differs between analogous simulations. The viscoplastic rheology enhances the upwelling and thermal regime around the ridge segments and transform fault. This increases the amount of melt focused across offsets and, around the transform fault, decreases the depth to the melting region.

Figures 8a and 8b show that the change in morphology of the melting region alters the delivery of melt to the ridge axis. In the case of a viscoplastic rheology melt is delivered to a point close to the transform fault. However, Figures 8b and 8c show that the neither the position of maximum melt delivery, or the redistribution distance ζ are changed much.

Figure 8d shows the effect of a viscoplastic rheology on axial depth differences for a range of transform fault lengths. The dashed curves show the difference in crustal thickness as a function of offset length. Assuming flexural support of ridge topography, these are converted into a difference in axial depth. For all nonzero offsets viscoplastic simulations predict greater asymmetry in crustal thickness and axial depth. This difference in asymmetry arises because the viscoplastic rheology increases the amount of melt focused across offsets (Figures 8a and 3a). The significance of this effect increases with offset, resulting in increasingly greater differences in predicted crustal thickness and axial depth.

5. Conclusion

By extending the 2-D model of asthenospheric flow and melting by Katz et al. [2004] into three dimensions, we confirm that plate-induced mantle dynamics can account for morphological changes observed along the global MOR system. Our model assumes that the mantle viscosity is high enough for asthenospheric flow to be effectively plate driven. Ridge migration perturbs asthenospheric flow, causing faster upwelling and enhanced melting beneath the plate that is leading with respect to the direction of ridge migration in the hot spot reference frame. Under reasonable assumptions of 3-D melt focusing, this melting asymmetry causes a difference in axial depth and crustal thickness of ridge segments separated by an offset.

The sense of mantle asymmetry predicted by our model is consistent with that seen in geophysical observations of MORs [Forsyth et al., 1998a; Panza et al., 2010]. Predictions of differences in axial depth across ridge offsets generated by our models describe the general trend and amplitude of global MOR data. The amplitude of the data can be fit with a widely used model of 3-D melt focusing [Sparks and Parmentier, 1991] and an asthenospheric thickness of 300 km. To smooth variations in crustal thickness close to transform faults, we assume that melt is redistributed along the ridge segment axis by melt flow through the porous mantle and cracks. Additional experiments that include a viscoplastic rheology predict enhanced mantle temperatures and upwelling rates around the ridge segments and transform fault. This leads to an increase in the predicted asymmetry in axial depth. Future models can further constrain shallow mantle and lithospheric processes that distribute melt at the ridge axis by using a parameterization of mantle melting and incompatible element behavior.

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